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On the structure and representations of the insertion-elimination Lie algebra. (English)

Zbl 1160.17018
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Summary: We examine the structure of the insertion-elimination Lie algebra on rooted trees introduced in *A. Connes* and *D. Kreimer* [Ann. Henri Poincaré 3, No. 3, 411–433 (2002; Zbl 1033.81061)]. It possesses a triangular structure $\mathfrak{g} = \mathfrak{n}_+ \oplus \mathbb{C} \cdot d \oplus \mathfrak{n}_-$, like the Heisenberg, Virasoro, and affine algebras. We show in particular that it is simple, which in turn implies that it has no finite-dimensional representations. We consider a category of lowest-weight representations, and show that irreducible representations are uniquely determined by a "lowest weight" $\lambda \in \mathbb{C}$. We show that each irreducible representation is a quotient of a Verma-type object, which is generically irreducible.

MSC:

17B65 Infinite-dimensional Lie (super)algebras
17B10 Representations of Lie algebras and Lie superalgebras, algebraic theory (weights)
17B66 Lie algebras of vector fields and related (super) algebras
17B81 Applications of Lie (super)algebras to physics, etc.

Keywords:

Lie algebras; Hopf algebras; pre-Lie relation; lowest weight representations; Verma modules; Connes-Kreimer algebra; insertion-elimination Lie algebra

Full Text: DOI arXiv

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