Yang, Yang; **Zhang, Jihui**

Existence of solutions for some fourth-order boundary value problems with parameters. (English) Zbl 1166.34012

Nonlinear Anal., Theory Methods Appl., Ser. A, Theory Methods 69, No. 4, 1364-1375 (2008).

The authors consider the boundary value problem (BVP)

$$u^{(4)}(t) + \eta u^{(2)}(t) - \xi u(t) = \lambda f(t, u(t)), \quad 0 < t < 1,$$

$$u(0) = u(1) = u^{(2)}(0) = u^{(2)}(1) = 0$$

with continuous nonlinearity $f : [0,1] \times \mathbb{R} \to \mathbb{R}$ and fixed η, ξ such that

$$\frac{\xi}{\pi^4} + \frac{\eta}{\pi^2} < 1, \ \xi \geq -\frac{\eta^2}{4}, \ \eta < 2\pi^2$$

and where $\lambda \in \mathbb{R}^+$ is a parameter. Using Green's function the authors provide a fixed point formulation of (BVP) for which they find an action functional and apply variational methods. The investigations of the equivalent variational formulation involve a square root operator of a suitable integral functional.

Depending on the assumptions on the nonlinear term f, they further obtain the values of λ for which (BVP) has at least one and at least two nontrivial solutions.

The proofs are based on variational techniques involving Morse theory and local linking. The paper is interesting since it shows the interplay between the topological and the variational methods.

Reviewer: Marek Galewski (Łódź)

MSC:

34B15	Nonlinear boundary value problems for ordinary differential equations	Cited in **1** Review
47N20	Applications of operator theory to differential and integral equations	Cited in **21** Documents
58E30	Variational principles in infinite-dimensional spaces	

Keywords:

boundary value problem; homological nontrivial critical point; Morse theory; local linking

Full Text: DOI

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