Connes, Alain

Cohomologie cyclique et foncteurs Ext^n. (French) Zbl 0534.18009

C. R. Acad. Sci., Paris, Sér. I 296, 953-958 (1983).

In his articles "Non-commutative differential geometry. I, II" (I.H.E.S. preprints), the author has defined and studied the cyclic cohomology $H^n_\lambda(\mathcal{A})$ of a C^*-algebra. The purpose of this note is to show how that cohomology may be obtained in terms of the Ext functor, where the "modules" in question are functors from a suitable small category.

To be specific, the author defines a category Λ whose objects Λ_n are indexed by the non-negative integers and whose morphisms $f \in Hom(\Lambda_n, \Lambda_m)$ are homotopy classes of continuous increasing maps $\phi : S^1 \to S^1$ which map the $(n+1)th$ roots of unity into the $(m+1)th$ roots of unity. A Λ- module is then a covariant functor from Λ into the category of Abelian groups, and, for a field k, a $k(\Lambda)$-module is a covariant functor from Λ into the category of vector spaces over k. He associates to each unital algebra $\mathcal{A}$ over k a $k(\Lambda)$-module $\mathcal{A}^\diamond$ whose objects are the n-fold tensor products of $\mathcal{A}$ with itself. A similar definition is made when $\mathcal{A}$ is a ring; in this case $\mathcal{A}^\diamond$ is a Λ-module. The author shows that for a k-algebra $H^n_\lambda(\mathcal{A})$ is just $Ext^n_{k(\Lambda)}(\mathcal{A}^\diamond, k^\diamond)$.

Reviewer: W.Moran

MSC:

18G15	Ext and Tor, generalizations, Künneth formula (category-theoretic aspects)
16E40	(Co)homology of rings and associative algebras (e.g., Hochschild, cyclic, dihedral, etc.)
46L05	General theory of C^*-algebras
46M05	Tensor products in functional analysis
16Exx	Homological methods in associative algebras

Keywords:

cyclic cohomology of C^*-algebra; Ext