Critical graphs, matchings and tours or a hierarchy of relaxations for the travelling salesman problem. (English) Zbl 0535.05038 Combinatorica 3, 35-52 (1983).

From the authors’ abstract: "A (perfect) 2-matching in a graph $G = (V, E)$ is an assignment of an integer 0, 1 or 2 to each edge of the graph in such a way that the sum over the edges incident with each node is at most (exactly) two. The incidence vector of a Hamiltonian cycle, if one exists in $G$, is an example of a perfect 2-matching. For $k$ satisfying $1 \leq k \leq |V|$, we let $P_k$ denote the problem of finding a perfect 2-matching of $G$ such that any cycle in the solution contains more than $k$ edges. We call such a matching a perfect $P_k$-matching. Then for $k < l$, the problem $P_k$ is a relaxation of $P_l$. Moreover if $|V|$ is odd, then $P_{|V|-2}$ is simply the problem of determining whether or not $G$ is Hamiltonian. A graph is $P_k$-critical if it has no perfect $P_k$-matching but whenever any node is deleted the resulting graph does have one. If $k = |V|$, then a graph $G = (V, E)$ is $P_k$-critical if and only if it is hypomatchable (the graph has an odd number of nodes and whatever node is deleted the resulting graph has a perfect matching).

We prove the following results: 1. If a graph is $P_k$-critical, then it is also $P_l$-critical for all larger $l$. In particular, for all $k$, $P_k$-critical graphs are hypomatchable. 2. A graph $G = (V, E)$ has a perfect $P_k$-matching if and only if for any $X \subseteq V$ the number of $P_k$-critical components in $G[V-X]$ is not greater than $|X|$. 3. The problem $P_k$ can be solved in polynomial time provided we can recognize $P_k$-critical graphs in polynomial time. In addition, we describe a procedure for recognizing $P_k$-critical graphs which is polynomial in the size of the graph and exponential in $k$.

Reviewer: J. Schwarze

MSC:
05C38 Paths and cycles
05C45 Eulerian and Hamiltonian graphs
94C15 Applications of graph theory to circuits and networks

Keywords:
perfect matching; 2-matching; Hamiltonian cycle; hypomatchable

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References:


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