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The Hilbert function of a reduced \mathbf{K} -algebra. (English) Zbl 0535.13012

J. Lond. Math. Soc., II. Ser. 28, 443-452 (1983).

Let $A = \bigoplus_{i \geq 0} A_i$ be a graded k -algebra of finite type (where $A_0 = k$ is a field and A is generated as a k -algebra by A_1). Such algebras are called standard G -algebras by *R. P. Stanley* [*Adv. Math.* 28, 57-83 (1978; [Zbl 0384.13012](#))]. The Hilbert function $\{b_i\}, i \geq 0$, of A is defined by $b_i = \dim_k A_i$. In 1927 Macaulay characterized those sequences which could be the Hilbert functions of a standard G -algebra. Such sequences are now called 0-sequences. In this paper we characterize those sequences which can be the Hilbert function of a reduced standard G -algebra. We show that the obvious necessary conditions on the sequence (namely that both the sequence and its first difference be 0-sequences) are also sufficient. We call such 0-sequences differentiable. Our proof gives an algorithm for constructing a reduced standard G -algebra from a differentiable 0- sequence.

We also investigate to what extent the geometry of $\text{Proj } A$ is determined by the Hilbert function of A .

MSC:

13E15 Commutative rings and modules of finite generation or presentation; number of generators

14A05 Relevant commutative algebra

16W50 Graded rings and modules (associative rings and algebras)

13D03 (Co)homology of commutative rings and algebras (e.g., Hochschild, André-Quillen, cyclic, dihedral, etc.)

13C15 Dimension theory, depth, related commutative rings (catenary, etc.)

13F20 Polynomial rings and ideals; rings of integer-valued polynomials

Cited in **7** Reviews
Cited in **73** Documents

Keywords:

Hilbert function of graded algebra; reduced standard G -algebra; differentiable 0-sequence

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