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Probability. Transl. from the Russian by R. P. Boas. (English) Zbl 0536.60001

Graduate Texts in Mathematics, 95. New York etc.: Springer-Verlag. XI, 577 p., 54 figs. DM 148.00; $ 57.50 (1984).

This book is based on a three-semester course of lectures given by the author for students of mathematics at Moscow state university. Generally speaking, it is a delicate problem which material to include into any course, and how to teach it. Both these problems are solved very successfully in the book under review. The author has chosen suitable material from probability theory, mathematical statistics and stochastic processes, and, which is essential, he has found probably the best form of presentation. Let us make now acquaintance with the contents. Following some good traditions the author begins his book in Chapter I with elementary probability theory. Using the so-called finite probabilistic models, he introduces the reader to some fundamental concepts: sample space, events, probability, conditional probability, independence, random variables (r.v-s), expectations, correlation. Special attention is paid to the Bernoulli scheme and the corresponding law of large numbers, normal and Poisson approximations to the binomial distribution. First knowledge with random walk, martingales and Markov chains is given, too.

Chapters II, III and IV are devoted to the systematic treatment of modern probability theory based on Kolmogorov's generally accepted axiomatics. The author introduces and considers in many details the following notions: σ-algebras, σ-additive probability measures and their representations, the Lebesgue integral, random variables and elements, conditional expectations with respect to a σ-algebra, Gaussian systems of r.v-s, and so on. He presents with complete proofs several fundamental results such as Kolmogorov's theorems of extension and existence of probability measures, Lebesgue and Fatou theorems on the limit behavior of the expectations of sequences of r.v-s. Also, he describes conditions for uniform integrability of a family of r.v-s, and considers the criteria for the following four kinds of convergence: in probability, with probability one, in mean of order p, in distribution. The weak convergence of probability distributions and the method of characteristic functions for proving limit theorems (C.L.T., L.L.N.) are studied. The author considers the one-dimensional version of Prokhorov's theorem on the equivalence of the relative compactness and the tightness of families of probability distributions.

For sequences of independent r.v-s he discusses the following fundamental laws which express properties with probability 1: zero-one law, theorem for three series, the strong law of large numbers, the law of iterated logarithm. Let us note that in these three chapters, II–IV, other notions are also introduced and discussed.

Next four chapters, V–VIII, are devoted to random processes with discrete time parameters (random sequences). Chapters V and VI treat stationary sequences (in strict and wide sense). Here the author introduces ergodicity and mixing properties and presents several results for them. Further on, he considers the following topics: orthogonal stochastic measures, stochastic integrals, spectral representation, Wold's expansion. A few important statistical problems are studied such as estimation of the covariance function and spectral density, extrapolation, interpolation and the filtering problem. The Kalman-Bucy filter and its generalizations are discussed in many details.

In chapter VII the author studies sequences of r.v-s that form martingales and establishes many fundamental results such as Doob's decomposition theorems, convergence theorems for submartingales and martingales, Wald's identites, inequalities. He investigates the sets of convergence of martingale-type sequences and presents conditions for absolute continuity and singularity of probability distributions. Then these results are used for studying the probability of the exit of a random walk on a curvilinear boundary. The C.L.T. for dependent r.v-s forming martingale-differences is established, too. Finally, chapter VII is devoted to the theory of discrete-time Markov chains with countable state space. Here the main attention is paid to the problems of classification of the states, and the asymptotic behavior of the distributions. The book concludes with useful and precise historical and bibliographical notes, list of references, index of symbols, and index of terms and authors.

It is clear that this book contains important and interesting results obtained through a long time period, beginning with the classical Bernoulli's law of large numbers, and ending with very recent results concerning convergence of martingales and absolute continuity of probability measures. Let us note especially that the great number of ideas, notions and statements in the book are well-motivated, explained in detail and illustrated by suitably chosen examples and a large number of exercises. Thus, the present book is a synthesis of all significant classical ideas and results, and many of the major achievements of modern probability theory. On the whole it is a welcome addition to mathematical literature and can become an indispensable textbook for courses in stochastics. The translation of this important book makes it now available for the English speaking reader.

Reviewer: J.Stoyanov

MSC:

60-01 Introductory exposition (textbooks, tutorial papers, etc.) pertaining to probability theory
60Axx Foundations of probability theory
60Exx Distribution theory
60Fxx Limit theorems in probability theory
60Gxx Stochastic processes

Keywords:

Kolmogorov's theorems of extension; existence of probability measures; relative compactness; tightness