**Jarchow, H.**

**Remarks on a characterization of nuclearity.** (English) Zbl 0537.46008

Arch. Math. 43, 469-472 (1984).

It is well-known that a locally convex space E is nuclear iff $\ell_p\{E\} = \ell_p(E)$ holds, algebraically and topologically, for some (all) $1 \leq p < \infty$. Here $\ell_p\{E\}$ resp. $\ell_p(E)$ denotes the space of all E-valued $\ell_p$-sequences resp. weak $\ell_p$- sequences, topologized in the usual fashion. This paper presents classes of non-nuclear locally convex spaces E such that $\ell_p\{E\}$ and $\ell_p(E)$ yet coincide as linear spaces. In fact, let E be the locally convex space obtained from supplying a given Banach space X with the locally convex topology generated by all seminorms $x \mapsto \|Tx\|$, T ranging over all bounded operators with domain X and range in some Hilbert space. Then X verifies Grothendieck's theorem [cf. *G. Pisier*: Ann. Inst. Fourier 28, No.1, 69-90 (1978; Zbl 0363.46019)] iff $\ell_p\{E\} = \ell_p(E)$ algebraically for all $1 \leq p < \infty$ (equivalently, for some $1 \leq p < 2$). Similarly, X is a Hilbert-Schmidt space [cf. the author in Rend. Circ. Mat. Palermo, II. Ser. Suppl. 2, 153-160 (1982; Zbl 0503.46014)] iff $\ell_p\{E\} = \ell_p(E)$ algebraically, for all (some) $2 \leq p < \infty$. But for any Banach space X, with E as above, $\ell_p\{E\} = \ell_p(E)$ as locally convex spaces iff dim X$< \infty$, $\forall 1 \leq p < \infty$.

**MSC:**

46A13 Spaces defined by inductive or projective limits (LB, LF, etc.)
46A11 Spaces determined by compactness or summability properties (nuclear spaces, Schwartz spaces, Montel spaces, etc.)
46A45 Sequence spaces (including Köthe sequence spaces)
47B10 Linear operators belonging to operator ideals (nuclear, $p$-summing, in the Schatten-von Neumann classes, etc.)



**Keywords:**

nuclear locally convex spaces; absolutely summing operators; Hilbert- Schmidt space

**Full Text:** DOI

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