Elbert, Árpád; Laforgia, Andrea

On the square of the zeros of Bessel functions. (English) Zbl 0541.33001

SIAM J. Math. Anal. 15, 206-212 (1984).

For $\nu \geq 0$, let $c \equiv c(\nu, k, \alpha)$ be the kth positive x-zero of

$$J_\nu(x)\cos\alpha - Y_\nu(x)\sin\alpha, \quad 0 \leq \alpha < \pi,$$

and let $c(\nu, k, \alpha)$ be continued analytically to the ν-interval $(\alpha/\pi - k, 0)$. The authors' main result, in a different notation, is that there exists κ_0, $0 < \kappa_0 < 1$, such that when $k - \alpha/\pi > \kappa_0$ we have $dc/d\nu > 1$ and $d^2(c^2)/d\nu^2 > 0$, $0 \leq \nu < \infty$. This implies that $j^2_{\nu k}$ is convex on $0 \leq \nu < \infty$ where $j_{\nu k} = c(\nu, k, 0)$, $k = 1, 2, \ldots$. The authors' method is to consider $c(\nu, k, \alpha)$ as the solution of

$$dc/d\nu = 2c \int_0^\infty K_0(2c \quad \sinh \quad t)e^{-2\nu t} dt,$$

which satisfies $c \to 0$ as $\nu \downarrow (\alpha/\pi - k)$. They show that $j^2_{\nu k}$ fails to be convex on $(-k, \infty)$ for $k = 2, 3, \ldots$ but they conjecture that $j^2_{\nu l}$ is convex on $(-1, \infty)$.

Reviewer: M.Muldoon

MSC:

33C10 Bessel and Airy functions, cylinder functions, ${}_0F_1$

Keywords:
zeros of Bessel functions; convex

Full Text: DOI