**Turaev, V. G.**; **Viro, O. Ya.**

**Intersections of loops in two-dimensional manifolds. II: Free loops.** (English. Russian original)

Zbl 0543.57007

Math. USSR, Sb. 49, 357-366 (1984); translation from Mat. Sb., Nov. Ser. 121(163), No. 3, 359-369 (1983).

This paper is a sequel to Part I by the first author [ibid. 106(148), 566-588 (1978; Zbl 0384.57004)]. The following problems are approached (A being a connected 2-manifold): Which is the least number of intersection and self-intersection points (multiplicities taken into account) of loops belonging to given homotopy classes of maps $S^1 \to A$? Special cases of these are: Under which conditions are homotopy classes of maps $S^1 \to A$ represented by nonintersecting loops? Under which conditions are such classes represented by simple loops?

The problems are formulated in three variants: (1) free homotopy classes; (2) classes $\alpha \in \pi_1(A, a)$ with $a \in IntA$; (3) classes $\alpha \in \pi_1(A, a)$ with $a \in \partial A$. Answers for (3) were given in Part I in terms of a bilinear map $\mathbb{Z}[\pi_1(A, a)] \times \mathbb{Z}[\pi_1(A, a)] \to \mathbb{Z}[\pi_1(A, a)]$; an analogous pairing is used in variant (2) to derive necessary conditions for the special cases. Their sufficiency is proved here as a consequence of more general results for variant (1). The same pairing is used to provide, first, lower bounds for the numbers of intersections and self-intersections of loops and, then, upper bounds for the minimal values of these numbers; in a large number of cases, the exact minimal values are derived.

Reviewer: J.Weinstein

**MSC:**

57N05 Topology of the Euclidean 2-space, 2-manifolds (MSC2010)
57M05 Fundamental group, presentations, free differential calculus

Cited in **3** Documents

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