Glover, Keith

The problem is considered of approximating a linear system, with transfer function matrix $G(s)$ of McMillan degree $n$, by a linear system with transfer function matrix $\hat{G}(s)$ of McMillan degree $k < n$. A complete characterization is derived of all approximations that minimize the Hankel norm of the error system: $\|G - \hat{G}\|_H$. The Hankel norm is an induced operator norm obtained by considering a linear system as a mapping between square integrable "past" inputs and square integrable "future" outputs. The key to the solution to this problem is the characterization of all matrices, with rational entries, in the form $\hat{G}(s) + F(s)$ which minimize

$$\|G - \hat{G} - F\| = \sup_{\omega} \sigma(G(j\omega) - \hat{G}(j\omega) - F(j\omega)),$$

where $\hat{G}(s)$ has McMillan degree $k$, $F(s)$ is anti-causal and $\sigma(.)$ denotes the maximum singular value of a matrix. The solution to this problem is obtained using results on balanced realizations, all-pass functions and the inertia of matrices. An algorithm is presented for finding Hankel norm approximations, and various error bounds on the approximation error are given. For one class of approximants it is shown that $\|G - \hat{G}\| < \sum_{i=k+1}^{n} \sigma_i(G)$, where $\sigma_i(G)$ is the $i$th largest Hankel singular value of $G$. Bounds such as this are important for predicting the performance of control schemes designed for $\hat{G}$ but applied to $G$.

Although the impetus for obtaining the results contained in this paper owes much to the pioneering work of V. M. Adamyan, D. Z. Arov and M. G. Krejn [Mat. Sb., Nov. Ser. 86(128), 34-75 (1971; Zbl 0243.47023); Izv. Akad. Nauk Arm. SSR, Mat. 6, 87-112 (1971; Zbl 0311.15012)] on the approximation of Hankel matrices, the author has developed his own rather ingenious methods of obtaining closed form solutions to the finite dimensional case. The paper is self-contained, and although rather long, is extremely well written and will reward its reader with many interesting results.

Reviewer: D.A.Wilson

MSC:

93C35 Multivariable systems, multidimensional control systems
15A60 Norms of matrices, numerical range, applications of functional analysis to matrix theory
41A20 Approximation by rational functions
15B57 Hermitian, skew-Hermitian, and related matrices
93C05 Linear systems in control theory
47A30 Norms (inequalities, more than one norm, etc.) of linear operators
93A15 Large-scale systems

Keywords:
transfer function matrix; Hankel norm; balanced realizations; all-pass functions; inertia of matrices

Full Text: DOI

References:
