Odlyzko, A. M.; **te Riele, Herman J. J.**
Disproof of the Mertens conjecture. (English) Zbl 0544.10047
J. Reine Angew. Math. 357, 138-160 (1985).

The Mertens conjecture states that $|M(x)| < x^{1/2}$ for all $x > 1$, where $M(x) = \sum_{n \le x} \mu(n)$, and μ (n) is the Möbius function. This conjecture has attracted a substantial amount of interest in its almost 100 years of existence because its truth was known to imply the truth of the Riemann hypothesis. This paper disproves the Mertens conjecture by showing that

$$\limsup_{x\to\infty} M(x)x^{-1/2} > 1.06, \quad \textit{and} \quad \liminf_{x\to\infty} M(x)x^{-1/2} < -1.009.$$

The disproof relies on extensive computations with the zeros of the zeta-function, and does not provide an explicit counterexample.

MSC:

11N37 Asymptotic results on arithmetic functions
11M06 $\zeta(s)$ and $L(s,\chi)$
11-04 Software, source code, etc. for problems pertaining to number theory

Keywords:

disproof; computation of zeros of Riemann zeta-function; Mertens conjecture; Möbius function

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