Let $G$ denote a groupoid, $< G >$ denotes the groupoid generated by $G$ under set product \[ e.g. GG^2 \in < G > \text{ where } GG^2 = \{a(bc) \mid a, b, c \in G\} \], $R(G)$ denotes the set of right ideals of $G$, and $P(G) = \{G^n \mid n \text{ is a positive integer}\}$ where $x^{n+1} = x^n x$ for $x \in G$. It is well known that if $G$ is a semigroup then: (i) $R(G)$ is a semigroup under set product, (ii) $< G > \subseteq R(G)$, (iii) $< G >$ is totally ordered under inclusion. In general (i), (ii), (iii) are not true. However, in this paper, it is shown that if $G$ is a right distributive groupoid \[ i.e. (xy)z = (xz)(yz) \text{ then } R(G) \text{ is a right distributive groupoid and conditions (ii) and (iii) are satisfied.} \]

Examples indicate that, in general, $P(G) \neq < G >$ and that $< G >$ is right distributive does not imply $G$ is right distributive (although the converse is true). The following are equivalent: (a) $< G >$ is right distributive; (b) if $Y, V \in < G >$ such that $Y \neq G$ then $YV = YG$ and $(GV)G = G^3$; if $A, B, C \in < G >$, then $(AB)C = (AG)G$. Finally if $< G >$ is right distributive the following conditions, on $< G >$, are characterized: $< G > = P(G)$, commutativity, associativity, distributivity (both sides).

MSC:

20M10 General structure theory for semigroups
20M12 Ideal theory for semigroups

Keywords:
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