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The basic gamma function $\Gamma_q(x)$ satisfies the recurrence relation $\Gamma_q(x+1)(1-q) = \Gamma_q(x)(1-q^x)$. By considering the logarithmic derivative of the basic gamma function g(x) which satisfies the recurrence relation $g(x+1) = g(x)(q^x \ln q)/(1-q^x)$, it is shown that g(x) cannot satisfy any algebraic differential equation of finite order. This same result also applies to the basic gamma function itself which establishes the required basic analogue of Hölder's theorem.

Reviewer: H.Exton

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33E99 Other special functions

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