As the title indicates, the author is concerned with the iterative solution of indefinite symmetric linear systems. In this case, if the linear system is written as \( Ax = f \), \( A \) is a large invertible symmetric matrix with both positive and negative eigenvalues. The author’s approach is to regard the spectrum of \( A \) as contained in the union of two disjoint intervals, \([a,b] \cup [c,d]\) with \( b < 0 < c \). A generalized Chebyshev semi-iterative method can then be defined by generating the set of polynomials orthogonal with respect to the weighted integral inner product given by
\[
\int_{a}^{b} p_1(\lambda) p_2(\lambda) w_1(\lambda) d\lambda + \int_{c}^{d} p_1(\lambda) p_2(\lambda) w_2(\lambda) d\lambda,
\]
where \( w_1(\lambda) \) and \( w_2(\lambda) \) represent the usual Chebyshev weight functions for \([a,b]\) and \([c,d]\) respectively. Similarities to conjugate gradient and conjugate residual methods are noted, and such algorithms are also discussed.

The paper is largely self-contained. The theory of polynomials orthogonal over two disjoint intervals is developed and algorithms are presented reflecting the theory. A priori error bounds are established in terms of the Chebyshev polynomials for \( A^2 \) over \([a,d]\) in order to prove convergence of the method. Estimates of the uniform norm of the orthogonal polynomials are also presented. An application to interior eigenvalue problems is also mentioned.

The practical problem of estimating the parameters \( a,b,c,d \) in an adaptive fashion is attacked. The extreme values \( a \) and \( d \) can be found using well-known methods such as the Lanczos algorithm. The interior parameters \( b \) and \( c \) can be estimated by iterating with a polynomial with fixed degree and then using Rayleigh-Ritz type estimates. Numerical experiments and comparison with the SYMMLQ algorithm of C. C. Paige and M. A. Saunders [ibid. 12, 617-629 (1975; Zbl 0319.65025)] are presented.

Reviewer: M. Sussman