

Benacerraf, Paul (ed.); Putnam, Hilary (ed.)

[Carnap, R.; Heyting, A.; von Neumann, J.; Brouwer, L. E. J.; Dummett, M.; Frege, G.; Russell, B.; Hilbert, D.; Curry, H. B.; Kreisel, Georg; Bernays, P.; Benacerraf, P.; Putnam, H.; Ayer, A. J.; Quine, W. V.; Hempel, C. G.; Poincaré, H.; Gödel, K.; Boolos, G.; Parsons, C.; Wang, Hao]

Philosophy of mathematics. Selected readings. 2nd ed. (English) [Zbl 0548.03002](#)

Cambridge etc.: Cambridge University Press. viii, 600 p. hbk: £30.00; \$ 39.50; pbk: £10.95; \$ 18.95 (1983).

The first edition of this collection of texts and articles (Prentice-Hall, 1964) has for two decades served as perhaps the best guide to the modern philosophy of mathematics. The second edition is substantially revised: 11 of the old papers have been replaced by 9 new ones, the Introduction by the Editors is reworked, and a handy collection of all separate bibliographies into one ends the volume. Thus, almost half of the book is new relative to the first edition.

Part I gives an excellent survey of the three main schools within the foundations of mathematics: logicism (Frege, Russell, Carnap), formalism (Hilbert, von Neumann, Curry, Kreisel), and intuitionism (Brouwer, Heyting, Dummett). The only change in this part is the addition of Dummett's very readable article 'The philosophical basis of intuitionistic logic' (1973).

Part II deals with ontology or the existence of mathematical objects. Quine's classical 'On what there is', which is easily accessible elsewhere, and Goodman's and Alston's papers about nominalism have been dropped, while Carnap's empiricist treatment of abstract entities and Bernays's discussion of Platonism have been retained. Two new articles have been added: Benacerraf's 'What numbers could not be' (1965), arguing that numbers are not objects at all, and Putnam's 'Mathematics without foundations' (1967). Here Putnam endorses a view - the truth value of some propositions in mathematics exists but is undiscoverable by us - which he questions in the new Introduction.

Part III discusses mathematical truth with the old articles by Ayer, Quine, Hempel, and Poincaré. Nagel's article on conventionalism is dropped, and Quine's well-known 'Two dogmas of empiricism' has been changed to his 'Carnap and logical truth' (1963). Gasking's and Castañeda's papers about the applicability of mathematics to the world have been replaced by Benacerraf's 'Mathematical truth' (1973) and Putnam's 'Models and reality' (1980). Here Putnam presents a model-theoretical argument - at present much debated among philosophers - against metaphysical realism and in favor of a sort of verificationism.

In the first edition, Part IV consisted of selections from Wittgenstein's 'Remarks on the Foundations of Mathematics' and three commentaries on Wittgenstein's constructivist position. In the new edition, Part IV is devoted to set theory. Gödel's two classical papers on Cantor and Russell are now joined with three new papers: Boolos's 'The iterative conception of set' (1971), Parsons's 'What is the iterative conception of set?' (1977), and Wang's 'The concept of set' (1974).

No single book can cover all the interesting aspects of the philosophy of mathematics - for example, the present one does not deal with the foundations of geometry, the implications of category theory, or the methodology and the heuristics of mathematical reasoning (Pólya, Lakatos). Still, it gives a comprehensive and balanced overview of the central issues within the ontology and the epistemology of mathematics, and nicely illustrates the vitality and the richness of this branch of philosophy.

Contents: Preface to the second edition; Introduction (pp. 1-37);

R. Carnap, The logicist foundations of mathematics (pp. 41-52; translation from *Erkenntnis* 2, 91-105 (German) (1931; [Zbl 0002.32101](#)));

A. Heyting, The intuitionist foundations of mathematics (pp. 52-61; translation from *Erkenntnis* 2, 106-115 (German) (1931; [Zbl 0002.32102](#)));

J. von Neumann: The formalist foundations of mathematics (pp. 61-65; translation from *Erkenntnis* 2, 116-121 (German) (1931; [Zbl 0002.32201](#)));

A. Heyting, Disputation (pp. 66-76; excerpted from "Intuitionism: an introduction" (1956; [Zbl 0070.00801](#)));

L. E. J. Brouwer, Intuitionism and formalism (pp. 77-89; repr. from *Bull. Am. Math. Soc.* 20, 81-96

- (1913; [JFM 44.0085.06](#)));
- L. E. J. Brouwer*, Consciousness, philosophy, and mathematics (pp. 90–96; excerpted from *Libr. 10th int. Congr. Philos.*, Amsterdam 1948, Vol. I, Proceedings, 1235–1249 (1949));
- M. Dummett*, The philosophical basis of intuitionistic logic (pp. 97–129; repr. from *Logic Colloquium 1973*, 5–40 (1975; [Zbl 0325.02005](#)));
- G. Frege*, The concept of number (pp. 130–159; translation from “Die Grundlagen der Arithmetik”, pp. 67–104, 115–119 (Breslau, 1884));
- B. Russell*, Selections from “Introduction to mathematical philosophy” (pp. 160–182; excerpted from the mentioned book, pp. 1–19, 194–206 (London, 1919; [JFM 47.0036.12](#)));
- D. Hilbert*, On the infinite (pp. 183–201; translation from *Math. Ann.* 95, 161–190 (German) (1925; [JFM 51.0044.02](#)));
- H. B. Curry*, Remarks on the definition and nature of mathematics (pp. 202–206; repr. from *Dialectica* 8, 228–233 (1954));
- G. Kreisel*, Hilbert’s programme (pp. 207–238; revised version of *Dialectica* 12, 346–372 (1958; [Zbl 0090.01004](#)));
- R. Carnap*, Empiricism, semantics and ontology (pp. 241–257; repr. from “Meaning and necessity”, pp. 205–221 (1947; [Zbl 0034.00106](#)));
- P. Bernays*, On platonism in mathematics (pp. 258–271; translation from *Enseign. Math.* 34, 52–69 (French) (1935; [Zbl 0014.00101](#)));
- P. Benacerraf*, What numbers could not be (pp. 272–294; repr. from *Philos. Review* 74, 47–73 (1965));
- H. Putnam*, Mathematics without foundations (pp. 295–311; repr. from *J. Philos.* 64, 5–22 (1967));
- A. J. Ayer*, The a priori (pp. 315–328; excerpted from “Language, truth and logic”, pp. 71–87 (London, 1956));
- W. V. Quine*, Truth by convention (pp. 329–354; corrected version of the paper in “Philosophical essays for A. N. Whitehead” (New York, 1936));
- W. V. Quine*, Carnap and logical truth (pp. 355–376; repr. from *Synthese* 12, 350–374 (1960; [Zbl 0161.00301](#)));
- C. G. Hempel*, On the nature of mathematical truth (pp. 377–393; repr. from *Am. Math. Mon.* 52, 543–556 (1945; [Zbl 0060.01909](#)));
- H. Poincaré*, On the nature of mathematical reasoning (pp. 394–402; excerpted from “Science and hypothesis”, pp. 1–9 (1952; [Zbl 0049.29106](#)));
- P. Benacerraf*, Mathematical truth (pp. 403–420; repr. from *J. Philos.* 70, 661–680 (1973));
- H. Putnam*, Models and reality (pp. 421–444; repr. from *J. Symb. Logic* 45, 464–482 (1980; [Zbl 0443.03003](#)));
- K. Gödel*, Russell’s mathematical logic (pp. 447–469; repr. of the paper in “The philosophy of Bertrand Russell”, pp. 125–153 (Evanston and Chicago, 1944));
- K. Gödel*, What is Cantor’s continuum problem? (pp. 470–485; revised and expanded version of *Am. Math. Mon.* 54, 515–525 (1947; [Zbl 0038.03003](#)));
- G. Boolos*, The iterative concept of set (pp. 486–501; repr. from *J. Philos.* 68, 215–232 (1971));
- C. Parsons*, What is the iterative conception of set? (pp. 503–529; repr. from *Logic, Found. Math., Comput. Theory*; *Proc. 5th int. Congr.*, London/Ontario 1975, Part 1, 335–367 (1977; [Zbl 0382.03008](#)));
- Hao Wang*, The concept of set (pp. 530–570; repr. from “From mathematics to philosophy”, pp. 181–223 (1974));
- Bibliography (pp. 571–600).

Reviewer: [Ilkka Niiniluoto \(Helsinki\)](#)

MSC:

- 03-03 History of mathematical logic and foundations
- 03A05 Philosophical and critical aspects of logic and foundations
- 00A30 Philosophy of mathematics
- 01A75 Collected or selected works; reprintings or translations of classics
- 00B10 Collections of articles of general interest
- 01A60 History of mathematics in the 20th century

Cited in **2** Reviews
Cited in **49** Documents

Keywords:

arithmetic; logicism; formalism; intuitionism; ontology; existence of mathematical objects; mathematical truth; set theory; philosophy of mathematics; epistemology