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Optimal recovery of linear functionals on sets of finite dimension. (English. Russian original)

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Let X be a linear space of dimension $n+1$ generated by f_0, f_1, \dots, f_n . Let L_1, \dots, L_n be linear functionals linearly independent defined on X and $L_0 \neq L_i, i = 1, \dots, n$, another linear functional on X . Let $P = \{\sum_{i=0}^n a_i f_i : a_i \in \mathbb{R}, |a_i| \leq |\beta_i|, i = 0, \dots, n\}$, where $\beta_n = 1$ and $(\beta_0, \dots, \beta_{n-1})$ is the solution of the system $\sum_{i=0}^n \beta_j L_j f_i = 0, j = 1, \dots, n$. The author presents an algorithm that recovers the functional L_0 on P with the least error among all linear algorithms using the information $(L_1 f, \dots, L_n f)$.

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MSC:

65D15 Algorithms for approximation of functions

65D05 Numerical interpolation

65Y20 Complexity and performance of numerical algorithms

Keywords:

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