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Conway products and links with multiple bridge surfaces. (English) Zbl 1158.57011

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For a given link in a 3-manifold, it is an interesting problem to compare two bridge surfaces. One can follow the program developed by *H. Rubinstein* and *M. Scharlemann* [Topology 35, No. 4, 1005–1026 (1996; Zbl 0858.57020)] to compare distinct Heegaard splittings of a given non-Haken 3-manifold. There the restriction to non-Haken manifolds was introduced to ensure that the relevant Heegaard splittings were strongly irreducible.

In the paper under review, under the analogous condition that the considered bridge surfaces are c -weakly incompressible, it is shown that, given two different bridge surfaces for a knot, either they can be properly isotoped to intersect in a nonempty collection of curves that are essential (including non-meridional) on both surfaces, or the knot is a Conway product with respect to an incompressible Conway sphere that naturally decomposes both surfaces into bridge surfaces for the respective factor link(s).

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MSC:

- 57M25 Knots and links in the 3-sphere (MSC2010)
- 57M27 Invariants of knots and 3-manifolds (MSC2010)
- 57M50 General geometric structures on low-dimensional manifolds

Cited in **1** Review
Cited in **2** Documents

Keywords:

bridge position; Heegaard splitting; strongly irreducible; weakly incompressible; Conway spheres

Full Text: [DOI](#) [arXiv](#)

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