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Nodal domains and spectral minimal partitions. (English) Zbl 1171.35083
Ann. Inst. Henri Poincaré, Anal. Non Linéaire 26, No. 1, 101-138 (2009).

Two-dimensional Schrödinger operators in bounded domains are considered. Results concerning the local properties of the nodal set of an eigenfunction are extended up to the boundary. For the two-dimensional case, it is shown that every open optimal partition is regular and strong. The relations between the nodal domains of eigenfunctions, spectral minimal partitions and spectral properties of the corresponding operator are analyzed. The existence and regularity results of the minimal partitions and the characterization of the minimal partitions, associated with nodal sets as the nodal domains of Courant-sharp eigenfunctions, represent the main result of the paper.

Reviewer: [Ruxandra Stavre \(București\)](#)

MSC:

[35P05](#) General topics in linear spectral theory for PDEs
[35J10](#) Schrödinger operator, Schrödinger equation
[35B65](#) Smoothness and regularity of solutions to PDEs

Cited in **4** Reviews
Cited in **56** Documents

Keywords:

[optimal partitions](#); [eigenvalues](#); [nodal domains](#); [spectral minimal partitions](#)

Full Text: [DOI](#) [EuDML](#) [arXiv](#)

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