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**Smallness problem for quantum affine algebras and quiver varieties.** (English) Zbl 1189.17014  
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A proper algebraic map  $\pi : Z \rightarrow X$  between irreducible complex algebraic varieties is *semi-small* if for a finite stratification of  $X$  into irreducible smooth subvarieties, the dimension of the inverse image of a point in a stratum is at most half the codimension of the stratum; it is *small* if in addition the equality holds only if the stratum is dense; [see *W. Borho* and *R. MacPherson*, *Astérisque* 101–102, 23–74 (1983; [Zbl 0576.14046](#))]. The interest of this notion stems from the elegant description of the singularities of a small morphism in terms of intersection cohomology sheaves. Fundamental examples of semi-small morphisms are the Springer resolution of the nilpotent cone and its partial versions. In [*H. Nakajima*, *Ann. Math.* (2) 160, No. 3, 1057–1097 (2005; [Zbl 1140.17015](#))] some varieties together with their resolutions were introduced, starting from a simply-laced Dynkin diagram with a fixed orientation. These are called graded quiver varieties and the resolutions are analogues of the Springer resolution. Nakajima translated the natural question of smallness of these resolutions into representation-theoretic terms, see loc. cit. Namely, let  $\mathcal{U}_q(\mathcal{L}(\mathfrak{g}))$  be the quantum loop algebra (the so-called Drinfeld realization), where  $\mathfrak{g}$  is the simple Lie algebra corresponding to the initial Dynkin diagram. Then the smallness of the resolutions of the graded quiver varieties is expressed in terms of characters of certain finite-dimensional  $\mathcal{U}_q(\mathcal{L}(\mathfrak{g}))$ -modules, called standard; those that satisfy the necessary condition for smallness of the resolution are called small.

The main result of the present paper is the characterization of the standard modules corresponding to Kirillov-Reshetikhin modules that are small. This is also extended to general simply-laced quantum affinizations.

Reviewer: [Nicolás Andruskiewitsch \(Cordoba\)](#)

**MSC:**

[17B37](#) Quantum groups (quantized enveloping algebras) and related deformations Cited in 9 Documents  
[14J17](#) Singularities of surfaces or higher-dimensional varieties

**Keywords:**

[quantum affine algebras](#); [graded quiver varieties](#)

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