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Limites et fonctions d'ensemble. (French) Zbl 0553.28008


In this paper a notion of limit for numerical functions with respect to a class of sets is introduced. More precisely, let X be a nonempty set; a function \( \beta : \mathcal{P}(X) \to \{0, 1\} \) is said an increasing set function if it assumes only the values 0 and 1, it is increasing, and \( \beta(\emptyset) = 0, \beta(X) = 1 \). For such a function it is possible to define \( \int f d\beta \) by \( \int f d\beta = \int f^+ d\beta - \int f^- d\beta \), where for a positive function g it is \( \int gd\beta = \int_0^{+\infty} \beta(g > t) dt \). A class \( \mathfrak{B} \subseteq \mathcal{P}(X) \) is said increasing if \( \mathfrak{B} \) is an increasing set function. For such classes it is possible to define \( \lim_{\mathfrak{B}} f \) by setting \( \lim_{\mathfrak{B}} f = \sup_{A \in \mathfrak{B}} \inf_A f \).

A functional \( T : \mathbb{R}^X \to \mathbb{R} \) is said to be an integral if i) \( f \leq g \Rightarrow T(f) \leq T(g) \); ii) for every \( \phi : \mathbb{R} \to \mathbb{R} \) continuous and increasing \( T(\phi f) = \phi(T(f)) \). The following characterizations hold: a) for every increasing class \( \mathfrak{B} \) we have \( \lim_{\mathfrak{B}} f = \int f d\mathfrak{B} \); b) for every integral \( T \) we have \( T(f) = \int f d\mathfrak{B} = \lim_{\mathfrak{B}} f \), where \( \beta(A) = T(1_A) \) and \( \mathfrak{B} = \{ A : T(1_A) = 1 \} \). Sections 2 and 3 of the paper are devoted to the relations between these definitions and de Giorgi’s \( \Gamma \)-limits [see E. De Giorgi: Boll. Unione Mat. Ital., V Ser., A 14, 213-220 (1977, Zbl 0389.49008)]. In section 4 many examples are presented.

Reviewer: G. Buttazzo

MSC:

28A25 Integration with respect to measures and other set functions
54C08 Weak and generalized continuity
49J99 Existence theories in calculus of variations and optimal control
54A20 Convergence in general topology (sequences, filters, limits, convergence spaces, nets, etc.)

Keywords:

\( \Gamma \) -convergence; limit for numerical functions with respect to a class of sets; increasing set function

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References:


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