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The shape genus of a shape map. (English) Zbl 0555.55011

Colloq. Math. 48, 35-47 (1984).

Let $f : X \to Y$ and $g : Z \to Y$ be maps of spaces and $\mathfrak{F}$ be a class of functors from the homotopy category of topological spaces into any other category. The author defines the $\mathfrak{F}$-genus of (f,g) to be the least integer $k \geq 1$ for which there are open sets V_m and maps $h_m : V_m \to X$, $1 \leq m \leq k$, such that $Z = \cup V_m$ and $f \circ h_m$ and $g \circ j_m$ are $\mathfrak{F}$-equal, where $j_m : V_m \to Z$ is inclusion (1$\leq m \leq k$); if no such integer exists, then the $\mathfrak{F}$-genus is ∞. Being $\mathfrak{F}$-equal means that $F([f \circ h_m)] = F([g \circ j_m)]$ for all $F \in \mathfrak{F}$. This concept of $\mathfrak{F}$-genus is said to generalize that of the genus of a map $f : X \to Y$ as in [*I. Berstein* and *T. Ganea*, Fundam. Math. 50, 265-279 (1962; Zbl 0192.293)].

An investigation is made of the dependency of $\mathfrak{F}$-genus on $\mathfrak{F}, f, g$. Then an extension is made so that $\mathfrak{F}$-genus can be defined when the maps are maps of inverse systems. This leads to a definition of shape $\mathfrak{F}$-genus for any pair $f : X \to Y$ and $g : Z \to Y$ of shape maps of spaces by taking it to be the genus of a pair of maps of inverse ANR-systems associated with f and g. If f,g are maps of CW-complexes, then the $\mathfrak{F}$-genus equals the shape $\mathfrak{F}$- genus.

The author states that in a forthcoming paper he will obtain necessary and sufficient conditions, involving the shape genus of certain maps, for an n-dimensional compactum to be embeddable up to shape into E^{2n}.

Reviewer: L.Rubin

MSC:

55P55 Shape theory
54C56 Shape theory in general topology
57N25 Shapes (aspects of topological manifolds)

Keywords:

pro-category; shape $\mathfrak{F}$-genus of shape maps; maps of inverse systems

Full Text: DOI