Cardaliaguet, P.; Lions, P.-L.; Souganidis, P. E.
A discussion about the homogenization of moving interfaces. (English) Zbl 1180.35070
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The authors present a number of new results concerning the behavior of moving interfaces in periodic environments and analyze in detail their averaging behavior. They concentrate on fronts moving by either first-order oscillatory velocity or mean curvature coupled with an oscillating forcing. Under sharp assumptions, they show that the fronts either homogenize or get trapped or oscillate. Several concrete examples are also discussed.

There has been considerable interest lately in the homogenization theory for first- and second-order partial differential equations in periodic/almost-periodic and stationary, ergodic, random environments. Of special interest is the study of the averaged behavior of moving interfaces in such environments. In this note the authors expand considerably this investigation in the context of periodic media. Although it is important both for applications and interesting mathematically to consider random media, very little is known for moving interfaces in this setting except for some first-order motions.

The general framework concerns the behavior as $\varepsilon \to 0$ of the solution u^ε of the initial value problem

$$\begin{cases} u_t^\varepsilon + F(\varepsilon D^2 u^\varepsilon, Du^\varepsilon, \frac{x}{\varepsilon}) = 0 & \text{in } \mathbb{R}^N \times (0,\infty), \\ u^\varepsilon = u_0 & \text{on } \mathbb{R}^N \times \{0\}, \end{cases}$$

where $u_0 \in UC(\mathbb{R}^N)$, the space of uniformly continuous functions on $\mathbb{R}^N$, and F is (degenerate) elliptic, geometric and periodic. In the random setting, the homogenization of this problem is still an open problem, except for completely degenerate (first-order) and (degenerate) semilinear equations with either convex or concave gradient dependence.

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MSC:

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35F21 Hamilton-Jacobi equations

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