Andres, Jan; **Kožušníková, Martina**; **Malaguti, Luisa**

Bound sets approach to boundary value problems for vector second-order differential inclusions. (English) Zbl 1177.34015

Nonlinear Anal., Theory Methods Appl., Ser. A, Theory Methods 71, No. 1-2, 28-44 (2009).

The paper deals with the second-order boundary value problem

$$\ddot{x}(t) \in F(t, x(t), \dot{x}(t)) \quad \text{for a.a. } t \in J, x \in S, \tag{1}$$

where $J = [t_0, t_1]$ is a compact interval, $F : J \times \mathbb{R}^n \times \mathbb{R}^n \multimap \mathbb{R}^n$ is an upper-Carathéodory mapping and S is a subset of $AC^1(J, \mathbb{R}^n)$. The authors develop a continuation principle for the solvability of (1) using fixed point index arguments. The main assumption which yields the possibility to apply the continuation principle is the transversality condition which is guaranted here by means of Liapunov-like bounding functions. In addition, for the Floquet semi-linear problem

$$\ddot{x}(t) + A(t)\dot{x}(t) + B(t)x(t) \in F(t, x(t), \dot{x}(t)) \quad \text{for a.a. } t \in J,$$

$$x(t_1) = Mx(t_0), \quad \dot{x}(t_1) = N\dot{x}(t_0),$$

where $A, B : J \to \mathbb{R}^n \times \mathbb{R}^n$ are integrable matrix functions and M and N are real $n \times n$ matrices with M non-singular, a viability result will be obtained by means of a bound sets technique.

Reviewer: Irena Rachůnková (Olomouc)

MSC:

34A60	Ordinary differential inclusions
34B15	Nonlinear boundary value problems for ordinary differential equations
47H04	Set-valued operators
47N20	Applications of operator theory to differential and integral equations

Keywords:

upper-Carathéodory differential incusions; Floquet problem; viability result

Full Text: DOI

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