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The proof of the Lane-Emden conjecture in four space dimensions. (English) Zbl 1171.35035
Adv. Math. 221, No. 5, 1409-1427 (2009).

The author considers the following Lamé-Emden system:

$$\begin{aligned} -\Delta u &= v^p, \\ -\Delta v &= u^q, \end{aligned}$$

in \mathbb{R}^n . The author proves that if $n = 3, 4$ and $\frac{1}{p} + \frac{1}{q} > 1 - \frac{2}{n}$, then the system above has no positive classical solutions. In the case $n \geq 5$ the author obtain a new region of nonexistence. The proof is based on Rellich-Pohozaev type identities, on a comparison property between components via the maximum principle, on Sobolev and interpolation inequalities on S^{n-1} and on feedback and measure arguments.

Reviewer: [Marco Biroli \(Milano\)](#)

MSC:

- 35J45 Systems of elliptic equations, general (MSC2000)
- 35J60 Nonlinear elliptic equations
- 35B33 Critical exponents in context of PDEs
- 35B45 A priori estimates in context of PDEs
- 35J50 Variational methods for elliptic systems

Cited in **105** Documents

Keywords:

[Nonlinear elliptic systems](#); [Nonexistence](#); [Liouville type theorems](#); [Lame-Emden conjecture](#)

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