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Topology of random clique complexes. (English) Zbl 1215.05163

Discrete Math. 309, No. 6, 1658-1671 (2009).

Summary: In a seminal paper, Erdős and Rényi identified a sharp threshold for connectivity of the random graph $G(n,p)$. In particular, they showed that if $p \gg \log n/n$ then $G(n,p)$ is almost always connected, and if $p \ll \log n/n$ then $G(n,p)$ is almost always disconnected, as $n \to \infty$.

The clique complex $X(H)$ of a graph H is the simplicial complex with all complete subgraphs of H as its faces. In contrast to the zeroth homology group of $X(H)$, which measures the number of connected components of H, the higher dimensional homology groups of $X(H)$ do not correspond to monotone graph properties. There are nevertheless higher dimensional analogues of the Erdős-Rényi Theorem.

We study here the higher homology groups of $X(G(n,p))$. For $k>0$ we show the following. If $p=n^\alpha$, with $\alpha < -1/k$ or $\alpha > -1/(2k+1)$, then the kth homology group of $X(G(n,p))$ is almost always vanishing, and if - $1/k < \alpha < -1/(k+1)$, then it is almost always nonvanishing.

We also give estimates for the expected rank of homology, and exhibit explicit nontrivial classes in the nonvanishing regime. These estimates suggest that almost all d-dimensional clique complexes have only one nonvanishing dimension of homology, and we cannot rule out the possibility that they are homotopy equivalent to wedges of a spheres.

MSC:

05C80 Random graphs (graph-theoretic aspects)
05C69 Vertex subsets with special properties (dominating sets, independent sets, cliques, etc.)

Keywords:

random graph; phase transition; clique complex; flag complex; discrete Morse theory

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