Da Prato, Giuseppe; Röckner, Michael; Wang, Feng-Yu

Singular stochastic equations on Hilbert spaces: Harnack inequalities for their transition semigroups. (English) Zbl 1193.47047

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The authors continue earlier investigations [*G. Da Prato* and *M. Röckner*, Probab. Theory Relat. Fields 124, No. 2, 261–303 (2002; Zbl 1036.47029)] in order to prove a Harnack inequality for solutions $(X(t))$ of stochastic differential equations (resp., their transition kernels) in the sense of *F.-Y. Wang* [Probab. Theory Relat. Fields 109, No. 3, 417–424 (1997; Zbl 0887.35012)] of the form

$$dX(t) = (AX(t) + F(X(t)))dt + \sigma dW(t), \quad X(0) = x(\in H),$$

where H is a separable Hilbert space, $(W(t))$ a cylindrical Brownian motion on H, σ a positive definite operator with bounded inverse, $(A, D(A))$ the generator of a C_0-one-parameter semigroup satisfying the growth condition $\langle Ax, x\rangle \leq \omega\|x\|^2$ on the domain $D(A)$, for some real ω. F is a set-valued m-dissipative map $F : H \supseteq D(F) \to 2^H$.

Let F_0 denote a map $F_0 : D(F) \to H$ satisfying $F_0(x) \in F(x)$ and $|F_0(x)| = \min_{y\in F(x)} |y|$. The corresponding Kolmogorov operator L_0, defined on a subspace $\mathcal{E}_A(H) \subseteq B_b(H)$, the space of bounded measurable real functions, is defined by

$$L_0(\varphi)(x) = \frac{1}{2}\mathrm{tr}(\sigma^2 D^2\varphi(x)) + \langle x, A^* D\varphi(x)\rangle + \langle F_0(x), D\varphi(x)\rangle$$

for $x \in D(F)$, $\varphi \in \mathcal{E}_A(H)$.

The investigations rely, as in the aforementioned paper, on several assumptions, $H_0 - H_5$. In particular, H_4 implies the existence of a infinitesimally invariant probability measure μ concentrated on the domain $D(F)$, and L_0 generates a Markov semigroup of transition kernels, called $p_t^\mu(\cdot, dx)$ (on $L^2(H,\mu)$), such that a Harnack inequality holds for $p > 1$, $f \in B_b(H)$ (Theorem 1.6):

$$(p_t^\mu f(x))^p \leq p_t^\mu f^p(y) \cdot \exp\left[\|\sigma^{-1}\|^2 p\omega|x-y|^2 / \left((p-1)(1-\mathrm{e}^{-2\omega t})\right)\right]$$

for $x, y \in \operatorname{supp}\mu =: H_0$ and $t > 0$.

The authors prove four corollaries of the main result, implying, e.g., the uniqueness of μ, estimates for the μ-densities of the kernels $p_t(y,\cdot)$ and hyper-boundedness of the transition operators, and, furthermore, $p_t^\mu(L^p(H,\mu)) \subseteq C(H_0)$ for all $t > 0$, hence the strong Feller property.

The proof runs along the following steps: first the measurable function F (resp., F_0) is approximated by resolvents $x \mapsto F_\alpha(x) := \frac{1}{\alpha}((I-\alpha F)^{-1} - I)(x)$ (Yosida approximation), $\alpha > 0$, which are single-valued Lipschitz functions, and these are approximated by C^∞-functions $F_{\alpha,\beta}$ (defined by regularizations with Gaussian distributions), and analogously, at first f is assumed to be bounded Lipschitz, then the results are extended to $f \in C_b(H)$, and finally to $f \in B_b(H)$.

Reviewer: Wilfried Hazod (Dortmund)

MSC:

47D07 Markov semigroups and applications to diffusion processes
60H10 Stochastic ordinary differential equations (aspects of stochastic analysis)
60J25 Continuous-time Markov processes on general state spaces
60J35 Transition functions, generators and resolvents

Keywords:

stochastic differential equations; Harnack inequality; Kolmogorov operator; infinitesimally invariant prob-

ability; invariant probability measure; Markov transition kernels; monotone coefficients; Yosida approximation

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