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Diffusions hypercontractives. (French) Zbl 0561.60080

Sémin. de probabilités XIX, Univ. Strasbourg 1983/84, Proc., Lect. Notes Math. 1123, 177-206 (1985).

[For the entire collection see Zbl 0549.00007.]

Let (P_t) be a Markovian semigroup of diffusion with infinitesimal generator L and stationary probability μ. The hypercontractivity of (P_t) means that there exists a constant $\lambda > 0$ such that for all $p \geq 1$, $q \geq 1$, $t > 0$, satisfying $q - 1 \leq (p-1)e^{\lambda t}$, $\|P_t f\|_{L^q} \leq \|f\|_{L^p}$, $f \in L^p(\mu)$. After proving some equivalent formulations of hypercontractivity, including Sobolev's logarithmic inequalities, in terms of Γ and Γ_2, called square field respectively iterated square field operators by the authors:

$$\Gamma(f,g) = [L(fg) - fL(g) - gL(f)],$$

$$\Gamma_2(f,g) = [L\Gamma(f,g) - \Gamma(Lf,g) - \Gamma(f,Lg)],$$

sufficient conditions for hypercontractivity are established. Some examples are discussed to illustrate the availability of these conditions. As useful tools, the operators Γ and Γ_2 are investigated and calculated for some cases.

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