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Elliptic polynomials orthogonal on the unit circle with a dense point spectrum. (English)

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Summary: We introduce two explicit examples of polynomials orthogonal on the unit circle. Moments and the reflection coefficients are expressed in terms of the Jacobi elliptic functions. We find explicit expression for these polynomials in terms of elliptic hypergeometric functions. We show that the obtained polynomials are orthogonal on the unit circle with respect to a dense point measure. We also construct corresponding explicit systems of polynomials orthogonal on the interval of the real axis with respect to a dense point measure. They can be considered as an elliptic generalization of the Askey-Wilson polynomials of a special type.

MSC:

33E05 Elliptic functions and integrals
33C47 Other special orthogonal polynomials and functions
42C05 Orthogonal functions and polynomials, general theory of nontrigonometric harmonic analysis
33E30 Other functions coming from differential, difference and integral equations

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