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Weisz, Ferenc

Pointwise summability of Gabor expansions. (English) Zbl 1185.42006

J. Fourier Anal. Appl. 15, No. 4, 463-487 (2009).

Author's abstract: A general summability method, the so-called θ-summability method is considered for Gabor series. It is proved that if the Fourier transform of θ is in a Herz space then this summation method for the Gabor expansion of f converges to f almost everywhere when $f \in L_1$ or, more generally, when $f \in W(L_1, \ell_\infty)$ (Wiener amalgam space). Some weak type inequalities for the maximal operator corresponding to the θ-means of the Gabor expansion are obtained. Hardy–Littlewood type maximal functions are introduced and some inequalities are proved for these.

Reviewer: Richard A. Zalik (Auburn University)

MSC:

42B08 Summability in several variables
42C15 General harmonic expansions, frames
42C40 Nontrigonometric harmonic analysis involving wavelets and other special systems
42A38 Fourier and Fourier-Stieltjes transforms and other transforms of Fourier type
46B15 Summability and bases; functional analytic aspects of frames in Banach and Hilbert spaces

Keywords:

Wiener amalgam spaces; Herz spaces; θ-summability; Gabor expansions; Gabor frames; time-frequency analysis; Hardy–Littlewood inequality

Full Text: DOI

References:

[1] Balan, R., Daubechies, I.: Optimal stochastic encoding and approximation schemes using Weyl-Heisenberg sets. In: Feichtinger, H.G., Strohmer, T. (eds.) Advances in Gabor Analysis. Applied and Numerical Harmonic Analysis, pp. 259–320. Birkhäuser, Boston (2002) · Zbl 1033.94001

[2] Benedetto, J.: Gabor representation and wavelets. In: Colella, D. (ed.) Commutative Harmonic Analysis. Contemp. Math., vol. 91, pp. 9–27. American Mathematical Society, Providence (1989)

[3] Bergh, J., Löfström, J.: Interpolation Spaces, an Introduction. Springer, Berlin (1976) · Zbl 0344.46071

[4] Butzer, P.L., Nessel, R.J.: Fourier Analysis and Approximation. Birkhäuser, Basel (1971) · Zbl 0217.42603

[5] Carleson, L.: On convergence and growth of partial sums of Fourier series. Acta Math. 116, 135–157 (1966) · Zbl 0144.06402 · doi:10.1007/BF02392815

[6] Daubechies, I.: Ten Lectures on Wavelets. SIAM, Philadelphia (1992) · Zbl 0776.42018

[7] Fefferman, C.: On the convergence of multiple Fourier series. Bull. Am. Math. Soc. 77, 744–745 (1971) · Zbl 0234.42008 · doi:10.1090/S0002-9904-1971-12793-3

[8] Feichtinger, H.G.: An elementary approach to Wiener's third Tauberian theorem for the Euclidean n-space. In: Symposia Math., Analisa Armonica, vol. XXIX, pp. 267–301. Cortona (1988)

[9] Feichtinger, H.G., Weisz, F.: Inversion formulas for the short-time Fourier transform. J. Geom. Anal. 16, 507–521 (2006) · Zbl 1101.42012

[10] Feichtinger, H.G., Weisz, F.: The Segal algebra S 0(\(\mathbb{R}\) d) and norm summability of Fourier series and Fourier transforms. Monatsh. Math. 148, 333–349 (2006) · Zbl 1130.42012 · doi:10.1007/s00605-005-0358-4

[11] Feichtinger, H.G., Weisz, F.: Wiener amalgams and pointwise summability of Fourier transforms and Fourier series. Math. Proc. Camb. Philos. Soc. 140, 509–536 (2006) · Zbl 1117.43001 · doi:10.1017/S0305004106009273

[12] Feichtinger, H.G., Weisz, F.: Gabor analysis on Wiener amalgams. Sampl. Theory Signal Image Process. 6, 129–150 (2007) · Zbl 1156.42304

[13] Feichtinger, H.G., Zimmermann, G.: A Banach space of test functions for Gabor analysis. In: Feichtinger, H.G., Strohmer, T.

(eds.) Gabor Analysis and Algorithms. Theory and Applications. Applied and Numerical Harmonic Analysis, pp. 123–170. Birkhäuser, Boston (1998). Chap. 3 · Zbl 0890.42008

[14] Fejér, L.: Untersuchungen über Fouriersche Reihen. Math. Ann. 58, 51–69 (1904) · Zbl 34.0287.01 · doi:10.1007/BF01447779

[15] Garcia-Cuerva, J., Herrero, M.-J.L.: A theory of Hardy spaces associated to the Herz spaces. Proc. Lond. Math. Soc. 69, 605–628 (1994) · Zbl 0831.42012 · doi:10.1112/plms/s3-69.3.605

[16] Grafakos, L.: Classical and Modern Fourier Analysis. Pearson Education, New Jersey (2004) · Zbl 1148.42001

[17] Grafakos, L., Lennard, C.: Characterization of L p (R n) using Gabor frames. J. Fourier Anal. Appl. 7, 101–126 (2001) · Zbl 0982.42017 · doi:10.1007/BF02510419

[18] Gröchenig, K.: Foundations of Time-Frequency Analysis. Birkhäuser, Boston (2001) · Zbl 0966.42020

[19] Gröchenig, K., Heil, C.: Gabor meets Littlewood-Paley: Gabor expansions in L p (R d). Stud. Math. 146, 15–33 (2001) · Zbl 0970.42021 · doi:10.4064/sm146-1-2

[20] Gröchenig, K., Heil, C., Okoudjou, K.: Gabor analysis in weighted amalgam spaces. Sampl. Theory Signal Image Process. 1, 225–259 (2002) · Zbl 1044.42025

[21] Heil, C.E., Walnut, D.F.: Continuous and discrete wavelet transforms. SIAM Rev. 31, 628–666 (1989) · Zbl 0683.42031 · doi:10.1137/1031129

[22] Herz, C.: Lipschitz spaces and Bernstein's theorem on absolutely convergent Fourier transforms. J. Math. Mech. 18, 283–324 (1968) · Zbl 0177.15701

[23] Hunt, R.A.: On the convergence of Fourier series. In: Orthogonal Expansions and their Continuous Analogues, Proc. Conf., Edwardsville, IL, 1967, pp. 235–255. Illinois Univ. Press, Carbondale (1968)

[24] Lu, S.: Four Lectures on Real H p Spaces. World Scientific, Singapore (1995) · Zbl 0839.42005

[25] Lyubarskii, Y., Seip, K.: Convergence and summability of Gabor expansions at the Nyquist density. J. Fourier Anal. Appl. 5, 127–157 (1999) · Zbl 0936.42016 · doi:10.1007/BF01261606

[26] Marcinkiewicz, J., Zygmund, A.: On the summability of double Fourier series. Fundam. Math. 32, 122–132 (1939) · Zbl 65.0266.01

[27] Stein, E.M.: Harmonic Analysis: Real-variable Methods, Orthogonality and Oscillatory Integrals. Princeton Univ. Press, Princeton (1993) · Zbl 0821.42001

[28] Stein, E.M., Weiss, G.: Introduction to Fourier Analysis on Euclidean Spaces. Princeton Univ. Press, Princeton (1971) · Zbl 0232.42007

[29] Trigub, R.M., Belinsky, E.S.: Fourier Analysis and Approximation of Functions. Kluwer Academic, Dordrecht (2004)

[30] Weisz, F.: Several dimensional {\(\theta\)}-summability and Hardy spaces. Math. Nachr. 230, 159–180 (2001) · Zbl 1014.42008 · doi:10.1002/1522-2616(200110)230:1<159::AID-MANA159>3.0.CO;2-L

[31] Weisz, F.: Summability of Multi-Dimensional Fourier Series and Hardy Spaces. Mathematics and Its Applications. Kluwer Academic, Dordrecht (2002) · Zbl 1306.42003

[32] Zygmund, A.: Trigonometric Series, 3th edn. Cambridge Press, London (2002) · Zbl 1084.42003