Rohrlich, David E.
On L-functions of elliptic curves and cyclotomic towers. (English) Zbl 0565.14006
Invent. Math. 75, 409-423 (1984).

Let f be a normalized new form of weight 2, character ψ, and level N. Let P be a finite set of primes not dividing N, and let X be the set of all primitive Dirichlet characters which are unramified outside P and infinity. For $\chi \in X$, let $L(s,f,\chi)$ be the L-function attached to f and χ. The main result in this paper is the following theorem:

For all but finitely many $\chi \in X$, $L(1,f,\chi) \neq 0$.

Two consequences of this result are given. One is a conjecture of *B. Mazur* and *P. Swinnerton-Dyer* [Invent. Math. 25, 1–61 (1974; Zbl 0281.14016)] to the effect that the p-adic L-function attached to a Weil curve over an abelian number field is not identically zero.

The other is as follows: Let E be an elliptic curve defined over $\mathbb{Q}$ with complex multiplication by the ring of integers of an imaginary quadratic number field, let P be a finite set of primes where E has good reduction. Let L be the maximal abelian extension of $\mathbb{Q}$ unramified outside P and infinity, and let $E(L)$ be the group of L-rational points on E. Then $E(L)$ is finitely generated.

This generalizes a result of *K. Rubin* and *A. Wiles* [Number theory related to Fermat's last theorem, Proc. Conf., Prog. Math. 26, 237–254 (1982; Zbl 0519.14017)]. If the conjectures of Taniyama-Weil and Birch-Swinnerton-Dyer hold, then the restriction to the case of complex multiplication can be removed.

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MSC:

11G40 L-functions of varieties over global fields; Birch-Swinnerton-Dyer conjecture
11R23 Iwasawa theory
14G10 Zeta functions and related questions in algebraic geometry (e.g., Birch-Swinnerton-Dyer conjecture)

Keywords:

modular forms; elliptic curve with complex multiplication; group of rational points; p-adic L-function

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