The q-shifted factorial is defined by \((x;q)_n = (1-x)(1-xq)\cdots(1-xq^{n-1})\). Andrews conjectured that the constant term in the Laurent polynomial

\[
\prod_{1 \leq i < j < n} \frac{(x_i/x_j; q)_{a_i}}{(x_j/x_i; q)_{a_j}}
\]

is

\[
(q; q)_{a_1+\ldots+a_n}/(q; q)_{a_1}\cdots(q; q)_{a_n},
\]

as an extension of an earlier conjecture of F. J. Dyson [J. Math. Phys. 3, 140–156 (1962; Zbl 0105.41604)] that was proved by J. Gunson [ibid. 3, 752–753 (1962; Zbl 0111.43903)] and K. G. Wilson [ibid. 3, 1040–1043 (1962; Zbl 0113.21403)] (the case \(q = 1\)). A combinatorial proof is given in the present paper. This is a major advance, and is one more indication that enumerative combinatorics has come of age, and should be learned by many people who could use it, as well as those who are developing it.

Reviewer: R. Askey

MSC:

33D15 Basic hypergeometric functions in one variable, \(r\phi_s\)

05A15 Exact enumeration problems, generating functions

Keywords:

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