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Distribution of arithmetical functions in the mean with respect to progressions (theorems of Vinogradov-Bombieri type). (Russian) Zbl 0566.10037

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The authors prove some generalizations of the Bombieri-Vinogradov theorem. Let

$$\sum(x,f,k,\ell)=\sum_{n\le x;n\equiv\ell(modk)} f(n)-(1/\phi(k))\sum_{n\le x;(n,k)=1} f(n),$$

$$\delta(x,f,k)=\max_{(\ell,k)=1}\max_{y\le x}|\sum(y,f,k,\ell)|,\quad \Delta(Q,f,E)=\sum_{k\le Q,k\in E}\delta(x,f,k)$$

F(k;E) be the characteristic function of the set E,

$$\Delta_1(Q,f,E)=\sum_{k\le Q}F(k,E)\max_{(\ell,k)=1}\max_{y\le x}|$$

$$\sum_{p\le y;p\equiv\ell(modk)} f(p)\log p-(1/\phi(k))\sum_{p\le y;p\nmid k} f(p)\log p|,$$

f*g(n)= $\sum_{dm=n} f(d)g(m)$, $\hat g(n)$ and $\Lambda_g(n)$ be functions such that $g*\hat g=e$ and $g(n)\log n=(g*\Lambda_g)(n)$ respectively; $g_u(n)=\sum_{dm=u;d>u}\hat g(d)g(m)$; $M(f,x)=\sum_{n\le x}f(n)$; $L=\log x$, $Q_i=Q_i(x)$ be monotone - increasing to ∞ functions. Let $\mu_\alpha(D)$ be the class of multiplicative functions such that $M(|f|^4,x)\ll xL^{4\alpha}$, $\alpha\ge 0$ and for all primitive characters mod d, $d\in D$, $d<L^c\ell$ we have $\sum_{z<p\le y}\chi_d^*(p)f(p)\log p\ll yL^{-B}\ell$, where $z=\exp(L^\theta)$, $\theta=1-(\log\log L/\log L)$, $y\le x$, C_1,B_1 are some constants, $D\subset z^+$. The following main results (among many other results) are obtained:

Theorem 1. Let $g(1)\ne 0$ and let (1) $M(|\hat g|^4,x)\ll xL^{4\alpha}, M(|g_n|^4,x)\ll xL^{4\alpha}\log^{4\beta}u$ for $u\ge 2$, $\alpha,\beta,\gamma>0$. We assume that there exists $Q_1(x)$ such that (2) $\Delta(Q_1,g,x)\ll xL^{-A}$ and (3) $\sum_{n\le x;(n,m)=1}\chi_q(u)\hat g(u)\ll x\exp(-L^\sigma)\log m$, $\sigma>0$ for all primitive characters χ mod q and all m, where $q\le L^c\ell$, $c_1>0$. Then, if $C\le\min\{A-\delta(\alpha+1)-1, A_2-2-\alpha-\gamma, C_1-\alpha-\gamma-2\}-\epsilon$, $\epsilon>0$, $0<\delta<1$, Q(x)$\le\min\{Q_1(xL^{-2A_2}\exp(L^\delta)$, $\sqrt{x}L^{-c-\alpha-\gamma-1}\log^{-\beta-1}L\}$, we have $\Delta(Q,\hat g,N)\ll xL^{-c}$.

Theorem 2. Let $g(1)\ne 0$ and let $g(n)$ be a function satisfying (1), (2) and (3) with $\Lambda_g(u)$ instead of $\hat g$(u). Let $M(|\Lambda_g|^4,x)\ll xL^{4\alpha}$. Then $\Delta(Q,\Lambda_q,N)\ll xL^{-c}$, where Q and c are the same as in Theorem 1.

Theorem 3. (1) Let $f(u)$ be a fully multiplicative function, $f\in M_\alpha(D)$, $\Delta(Q_1,f,E)\ll xL^{-A}$. Then $\Delta_1(Q,f,E)\ll xL^{-A+1}\log^{\alpha+1}L$, where Q(x)$\le\min(Q_1(xL^{-2A-4\alpha-7/2}\log^{-2\alpha-26}L), \sqrt{x}L^{-A-2\alpha-3/4}\log^{-\alpha-13}L)$ and E is a subset of natural numbers having all its divisors $\in D$. (2) If $Q_1(x)\ge x^{1/2+\epsilon}$ and $f(u)=O(1)$, then $\Delta_1(Q,f,E)\ll xL^{-A+2}$ for $Q=\sqrt{x}L^{-A+1}\log^{-3/2}L$. If $f(u)=1$, then for $Q=\sqrt{x}L^{-A-1}\log^{-3/2}L$ we have

$$\sum_{k\le Q}\max_{(\ell,k)=1}\max_{y\le x}|\psi(y,k,\ell)-\frac{y}{\phi(k)}|\ll xL^{-A},$$

Theorem 4. Let $f\in M_\alpha(D)$ and $\Delta_1(Q,f,E)\ll xL^{-3B}$. Then

$$\Delta(Q_1,f,E)<\ll xL^{-B+5/6+4/3\alpha}Log^{\alpha+2}L$$

with $Q_1=\min(Q(x),\sqrt{x}L^{-3B-3/2-2\alpha}Log^{-5/4}L)$. Some interesting corollaries are also proved.

Reviewer: G.A.Kolesnik

MSC:

11N37 Asymptotic results on arithmetic functions
11N13 Primes in congruence classes

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generalizations of the Bombieri-Vinogradov theorem; multiplicative functions; fully multiplicative function

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