

**Read, C. J.**

**A solution to the invariant subspace problem.** (English) [Zbl 0566.47003](#)  
*Bull. Lond. Math. Soc.* 16, 337-401 (1984).

In studying the action of a matrix of a finite dimensional linear space, the existence of eigenvectors enables one to put the matrix in upper triangular form. Although the understanding is not nearly so complete for compact operators on Banach spaces, an analogous decomposition is possible, where eigenvectors must be replaced by invariant subspaces. For arbitrary bounded linear operators on a Banach space, one couldn't even begin since the existence problem for invariant subspaces was unsolved.

Most effort on this problem has been directed toward operators on Hilbert space and many papers have been written on results for special classes of operators. Still there was no known example of an operator having no proper invariant subspace. In [*Ann. Math.*, II. Ser. 117, 669-694 (1983; [Zbl 0553.47002](#))] *A. Atzmon* exhibited a continuous operator on a nuclear Fréchet space with no proper invariant subspace. Earlier *P. Enflo* claimed that he had constructed an operator on a Banach space with no proper invariant subspace but no proof has been published. In this paper *C. J. Read* constructs an example of a bounded linear operator on a nonreflexive Banach space with no proper invariant subspace. The example is complicated and not a "natural" operator. The author has since given a much simpler example defined on the Banach space  $\ell^1$ . The author's construction is ingenious and highly nontrivial but not very illuminating. But all in all this is a very nice piece of work!

Reviewer: [R.G.Douglas](#)

**MSC:**

[47A15](#) Invariant subspaces of linear operators

Cited in **9** Reviews  
Cited in **41** Documents

**Keywords:**

existence problem for invariant subspaces; example of a bounded linear operator on a nonreflexive Banach space with no proper invariant subspace

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