Brewster, M. E.; Kannan, R.
Nonlinear successive over-relaxation. (English) Zbl 0566.65045
Numer. Math. 44, 309-315 (1984).

Let Φ be a real strictly convex functional defined and twice continuously differentiable on a convex domain in $\mathbb{R}^n$. To find its minimum the authors solve a system of nonlinear equations $F(x) = 0$ in $\mathbb{R}^n$, where F denotes grad Φ. They present two theorems giving sufficient conditions of convergence for: (i) a nonlinear analogue of the Gauss-Seidel method for positive-definite matrices, (ii) a nonlinear successive overrelaxation method, used for such a system of equations. The theorems are parallel to results of Schechter (1962, 1968) but give more general sufficient conditions and may be applied for a more general class of functionals whose Hessian matrix may be singular.

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MSC:

65K05 Numerical mathematical programming methods
90C25 Convex programming
65H10 Numerical computation of solutions to systems of equations

Keywords:

nonlinear successive over-relaxation; Gauss-Seidel; convergence

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