Chawla, M. M.; Shivakumar, P. N.

Numerov's method for non-linear two-point boundary value problems. (English)

Zbl 0566.65064
Int. J. Comput. Math. 17, 167-176 (1985).

The authors consider the application of Newton's method for solving the resulting nonlinear system in Numerov's method applied to nonlinear two- point boundary value problem of the form $y'' + f(x, y) = 0$, $0 \le x \le 1$, $y(0) = \alpha$, $y(1) = \beta$. The application of Newton's method given earlier by *P. Henrici* [Discrete variable methods in ordinary differential equations (1962; Zbl 0112.349)] and *M. Lees* [Numer. Solution Partial Diff. Equations, Proc. Sympos. Univ. Maryland 1965, 59- 72 (1966; Zbl 0148.392)] has not contained any clue as to the starting vector to be supplied. In this paper the authors propose a suitable initial approximation for use with Newton's method. Moreover, they present sufficient conditions guaranteeing convergence of Newton's method with this initial approximation for all $-\infty < \partial f/\partial y < \pi^2$. The authors consider the cases $-\infty < \partial f/\partial y \le 0$ and $0 < \partial f/\partial y < \pi^2$ separately, but in each case the speed of convergence in Newton's method is given by the same estimation. It is interesting that the initial approximation is based only on the boundary data. Therefore, it is possible to build up an automatic subroutine for solving nonlinear two-point boundary value problems by Numerov's method. At the end of this paper two numerical examples are presented. These examples entirely confirm the theoretical results.

Reviewer: A.Marciniak

MSC:

65L10 Numerical solution of boundary value problems involving ordinary differential equations
34B15 Nonlinear boundary value problems for ordinary differential equations

Keywords:

finite difference equations; Newton's method; Numerov's method; convergence; numerical examples

Full Text: DOI

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