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Rational approximations of the number $\sqrt[3]{3}$. (English. Russian original) Zbl 1205.11082

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Apparently, the best known irrationality measure of the number $3^{1/3}$ is 2.69267 (due to G. Chudnovsky). This means that the inequality $|q3^{1/3} - p| > q^{-1.69267}$ holds for each sufficiently large positive integer q and each integer p . In this paper the author shows that the constant 1.69267 can be replaced by the constant 1.50308 for a special sequence $q = 2^k$, $k \in \mathbb{N}$, namely, the inequality $\|3^{1/3}2^k\| > 0.3568^k$ holds for each sufficiently large positive integer k . The proof uses Padé approximants and is effective. The improvement compared to the general estimate comes from the fact the the powers of 2 naturally appear in the denominators q_n of corresponding ‘small’ linear forms $L_n = 3^{1/3} - p_n/q_n$.

Reviewer: [Artūras Dubickas \(Vilnius\)](#)

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11J04 Homogeneous approximation to one number

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Padé approximant; effective rational approximation; Laplace method

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