**Broutin, Nicolas**; **Devroye, Luc**; **McLeish, Erin**
**Note on the structure of Kruskal's algorithm.** (English) Zbl 1219.05181
Algorithmica 56, No. 2, 141-159 (2010).

Let $G=(V,E)$ be a connected edge-weighted graph and let $(V,F)$ be its minimal spanning tree constructed by Kruskal's algorithm [*J.B. Kruskal jun.*, "On the shortest spanning subtree of a graph and the traveling salesman problem," Proc. Am. Math. Soc. 7, 48–50 (1956; Zbl 0070.18404)]. We capture the evolution of the spanning forest from $(V,\emptyset)$ to $(V,F)$ by a rooted binary tree $R$ with leaves in $V$ and internal nodes in $F$. Let $h(G)$ denote the height of $R$. In case of Prim's algorithm we would have $h(G_n)=n-1$ for every connected graph $G_n$ on $n$ vertices. In case of Kruskal's algorithm there is a constant $c>0$ such that the probability of $h(G_n)\geq cn$ tends to 1 for $n\to\infty$, and therefore the expected value of $h(G_n)$ is in $\Theta(n)$, for three choices of random edge-weights:

(1) $G_n$ is a complete graph on $n$ independently uniformly distributed random points in $[0,1]^d$ and the edges are weighted by the Euclidean distance,

(2) $G_n$ is a complete graph on $n$ vertices and the edge-weights are independently uniformly distributed in $[0,1]$,

(3) $G_n$ is the Cartesian product of $d$ paths $P_k$, $n=k^d$, and the edge-weights are independently uniformly distributed in $[0,1]$.

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## MSC:

05C85 Graph algorithms (graph-theoretic aspects)
05C80 Random graphs (graph-theoretic aspects)
68R10 Graph theory (including graph drawing) in computer science

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random tree; minimal spanning tree; Kruskal; height; random graph; percolation

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