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Homotopy theory of simplicial sheaves in completely decomposable topologies. (English)

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The homotopy theory of simplicial (pre)sheaves introduced by Joyal and developed by *J. F. Jardine* [J. Pure Appl. Algebra 47, 35–87 (1987; Zbl 0624.18007), Can. J. Math. 39, 733–747 (1987; Zbl 0645.18006)] provides us with a proper simplicial model structure on the category of simplicial sheaves and presheaves and allows us to give definitions of all the important objects of homotopical algebra in the context of sheaves on all sites. As a consequence of its generality, some of the important classes of objects and morphisms have a very abstract definition. On the other hand, the Brown-Gersten approach [*K. S. Brown* and *S. M. Gersten*, Algebr. K-Theory I, Proc. Conf. Battelle Inst. 1972, Lect. Notes Math. 341, 266–292 (1973; Zbl 0291.18017)] for simplicial sheaves on Noetherian topological spaces of finite dimension is much more explicit, and gives a finitely generated model structure on the category of simplicial sheaves, but does not generalize to arbitrary sites.

The author of the paper under review defines a class of sites for which a generalized analog of the Brown-Gersten approach works. He proves that for such sites the class of weak equivalences of simplicial presheaves can be generated by an explicitly given set of generating weak equivalences.

Reviewer: J. Remedios (La Laguna)

MSC:

55U35 Abstract and axiomatic homotopy theory in algebraic topology
18F20 Presheaves and sheaves, stacks, descent conditions (category-theoretic aspects)
54B40 Presheaves and sheaves in general topology

Keywords:

simplicial sheaves

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References:

[1] Brown, K.S.; Gersten, S.M., (), 266-292
[2] Hirschhorn, Philip˜S., ()
[3] Hovey, Mark, ()
[4] Hovey, Mark, Spectra and symmetric spectra in general model categories, J. pure appl. algebra, 165, 1, 63-127, (2001) · Zbl 1008.55006
[5] Jardine, J.F., Simplicial objects in a Grothendieck topos, Contemp. math., 55, 1, 193-239, (1986) · Zbl 0606.18006
[6] Jardine, J.F., Simplicial presheaves, J. pure appl. algebra, 47, 35-87, (1987) · Zbl 0624.18007
[7] Jardine, J.F., Stable homotopy theory of simplicial presheaves, Canadian J. math., 39, 3, 733-747, (1987) · Zbl 0645.18006
[8] A. Joyal, Letter to A. Grothendieck
[9] Mac Lane, Saunders; Moerdijk, Ieke, Sheaves in geometry and logic, (1994), Springer-Verlag · Zbl 0822.18001
[10] Morel, Fabien; Voevodsky, Vladimir, $\mathbf{A}^1$-homotopy theory of schemes, Inst. hautes études sci. publ. math., 90, 2001, 45-143, (1999) · Zbl 0983.14007
[11] Voevodsky, Vladimir, Unstable motivic homotopy categories in nisnevich and cdh-topologies, J. pure appl. algebra, (2009) · Zbl 1187.14025
[12] Vladimir Voevodsky, Simplicial radditive functors, Journal of K-theory (2010) (in press) arXiv:0805.4434 · Zbl 1194.55021