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Convexity of measures in certain convex cones in vector space σ- algebras. (English)

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For $0 < \vartheta < 1$, $-\infty \leq a \leq \infty$ and $0 < s,t \leq \infty$ set $M_a^\vartheta = (s,t) = (\vartheta s^a + (1-\vartheta)t^a)^{1/a}$ if $a \in R \setminus \{0\}$, $= \min(s,t)$ if $a = -\infty$, $= s^\vartheta t^{1-\vartheta}$ if $a = 0$, and $= \max(s,t)$ if $a = \infty$ ($0^\alpha = \infty$ if $-\infty < \alpha < 0$). For $0 \leq s,t \leq \infty$ set $M_a^\vartheta(s,t) = 0$ if $s = 0$ or $t = 0$. Let E be a real locally convex space and C a closed convex cone in E (with vertex at 0); set $< C >= \{K - C : E \supset K \quad compact\}$. A finite positive Radon measure μ on E is said to be :α :-concave in C ($-\infty \leq \alpha \leq \infty$) if $\mu(\vartheta A + (1-\vartheta)B) \geq M_\alpha^\vartheta(\mu(A), \mu(B))$ for all $A, B \in< C >$ and $\vartheta \in (0,1)$. The author's study of :α :-concave measures is motivated by their application in reliability theory, statistics, stochastic programming, convexity, absolute continuity of semi-norms etc. and by the possibility of deepening of the Brunn-Minkowski theory on vector spaces. For example, the author derives zero-one laws and integrability of some sublinear functions, and presents examples of stochastic processes with increasing path inducing :0:-concave measures in suitable convex cones.

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MSC:

46A55 Convex sets in topological linear spaces; Choquet theory
60F20 Zero-one laws
46G12 Measures and integration on abstract linear spaces

Keywords:

closed convex cone; finite positive Radon measure; :α :-concave measures; Brunn-Minkowski theory; zero-one laws; integrability of some sublinear functions; stochastic processes

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