Chen, Yijia; **Flum, Jörg**

On the complexity of Gödel's proof predicate. (English) Zbl 1201.03050

J. Symb. Log. 75, No. 1, 239-254 (2010).

In the paper under review several algorithmic problems have been studied from the complexity-theoretic viewpoint. Most notable of them is the problem called p-Gödel, which asks, for a given natural number n in unary as input and a first-order sentence φ as parameter, whether φ has a proof of length $\leq n$. The main result of the paper is that the problem p-Gödel is not fixed-parameter tractable if $\mathbf{P}[\mathrm{TC}] \neq \mathbf{NP}[\mathrm{TC}]$. More precisely, the problem p-Gödel takes a first-order sentence φ and a natural number n in unary as input, considers the length $|\varphi|$ of φ as parameter, and asks whether φ has a proof of length $\leq n$. A parameterized problem is a pair (Q, κ) where $Q \subseteq \{0,1\}^*$ and $\kappa : \{0,1\}^* \to \mathbb{N}$; the function κ is required to be polynomial-time computable, and then the parameterized problem is the membership problem $x \in^? Q$. A parameterized problem (Q, κ) is called fixed-parameter tractable if the problem $x \in Q$ is solvable by an algorithm running in time $f(\kappa(x)) \cdot |x|^{O(1)}$ for a computable function f. And the notation $\mathbf{P}[\mathrm{TC}] \neq \mathbf{NP}[\mathrm{TC}]$ means that for all time constructible and increasing functions h one has $\mathrm{DTIME}(h^{O(1)}) \neq \mathrm{NTIME}(h^{O(1)})$. Let us note that if $\mathbf{P} = \mathbf{NP}$ then the equality $\mathrm{DTIME}(h^{O(1)}) = \mathrm{NTIME}(h^{O(1)})$ holds for every time constructible and increasing function $h : \mathbb{N} \to \mathbb{N}$. The paper also includes the study of some other related problems, with a pleasing flow of arguments. Familiarity with parameterized complexity theory is a prerequisite for reading the paper.

Reviewer: Saeed Salehi (Tabriz)

MSC:

03F20 Complexity of proofs

Cited in **2** Documents

Keywords:

proof length; parametrized Gödel problem; fixed-parameter tractable; parametrized complexity theory

Full Text: DOI

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