Bank, R. E.; **Weiser, A.**
Some a posteriori error estimators for elliptic partial differential equations. (English)
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Math. Comput. 44, 283-301 (1985).

The paper presents three new a posteriori error estimators in the energy norm for finite element solutions of certain linear, self-adjoint, positive-definite Neumann problems in R^2. The estimators are based on solving a local Neumann problem in each element. However, to be well posed the right side for these problems must be consistent in the limit when the mesh parameter h tends to zero. The three algorithms differ in the way in which this consistency condition is satisfied. If u_H and U denote the exact solution and the finite element approximation, respectively, then the error estimators $|||\bar{e}|||$ satisfy $(1-\epsilon_1)\|u_H-U\| \leq |||\bar{e}||| \leq (1+\epsilon_2)\|u_H-U\|$ where $\epsilon_1 < 1$ and ϵ_2 is bounded. Under certain conditions it is shown that ϵ_1 tends to zero. Numerical results are presented where the error estimators are upper bounds on the norm of the true error and are always within a factor of three of the norm of the true error. Moreover, one of the estimators appears to converge to the norm of the true error. The algorithms extend to some more general problems including problems where homogeneous Dirichlet boundary conditions are specified on all or part of the boundary.

Reviewer: W.C.Rheinboldt

MSC:

65N30 Finite element, Rayleigh-Ritz and Galerkin methods for boundary value problems involving PDEs
65N15 Error bounds for boundary value problems involving PDEs
35J25 Boundary value problems for second-order elliptic equations
65N50 Mesh generation, refinement, and adaptive methods for boundary value problems involving PDEs

Keywords:
finite element method; numerical examples; a posteriori error estimators; Neumann problems

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