Harui, Takeshi; **Kato, Takao**; **Komeda, Jiryo**; **Ohbuchi, Akira**
Quotient curves of smooth plane curves with automorphisms. (English) Zbl 1192.14028
Kodai Math. J. 33, No. 1, 164-172 (2010).

Let C be a smooth plane curve over $\mathbb{C}$ of degree $d \geq 4$ and suppose that σ is an automorphism of C of order $n \geq 2$ with number of fixed points equal to f. Then σ extends to an automorphism $\tilde{\sigma}$ of $\mathbb{P}^2$ which can be represented by a 3×3 diagonal matrix with either two 1's on the diagonal (called a type 1 automorphism) or a single 1 on the diagonal (called a type 2 automorphism). In the paper under review, the authors determine some properties of the quotient curve $B = C/\langle\sigma\rangle$, the automorphism σ and the degree d for type 1 and 2 automorphisms.

For type 1 automorphisms, the authors prove a number of extensive results which hold for any n. For example, the authors show that $d \equiv 0$ or $1 \mod (n)$ and moreover when $d \equiv 0 \mod (n)$ we have $f = d$, and when $d \equiv 1 \mod (n)$, $f = d + 1$. Additionally they show that the quotient curve B is isomorphic to a curve of degree d in $\mathbb{P}^{n+1}$ attaining the largest geometric genus given by the Castelnuovo bound. They also prove a converse statement. Specifically, given a curve B satisfying the properties derived, and integers $n \geq 2$ and $d \equiv 0$ or $1 \mod (n)$, there always exists a smooth plane curve C and an automorphism σ of order n of type 1 and a cyclic covering $\pi : C \to B = C/\langle\sigma\rangle$.

For type 2 automorphisms, the authors are able to provide similar partial results in the special case that $n = p$ a prime. Specifically, they show that either $f = 0$ and $d \equiv 0 \mod (p)$, $f = 2$ and $d \equiv 1$ or $2 \mod (p)$, or $f = 3$ and $d^2 - 3d + 3 \equiv 0 \mod (p)$ and $p \equiv 1 \mod (6)$ or $p = 3$. They finish by giving specific examples to show that each of these three cases occur.

Reviewer: Aaron Wootton (Portland)

MSC:

14H51 Special divisors on curves (gonality, Brill-Noether theory)

Cited in **1** Review
Cited in **2** Documents

Keywords:

plane curves; automorphisms; covering; quotient curves; linear system

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