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A bijective proof of a major index theorem of Garsia and Gessel. (English) Zbl 1189.05010
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Summary: In this paper we provide a bijective proof of a theorem of Garsia and Gessel describing the generating function of the major index over the set of all permutations of $[n] = \{1, \dots, n\}$ which are shuffles of given disjoint ordered sequences π_1, \dots, π_k whose union is $[n]$. The proof is based on a result (an “insertion lemma”) of Haglund, Loehr, and Remmel which describes the change in major index resulting from the insertion of a given new element in any place in a given permutation. Using this lemma we prove the theorem by establishing a bijection between shuffles of ordered sequences and a certain set of partitions. A special case of Garsia and Gessel’s theorem provides a proof of the equidistribution of major index and inversion number over inverse descent classes, a result first proved bijectively by Foata and Schützenberger in 1978. We provide, based on the method of our first proof, another bijective proof of this result.

MSC:

05A05 Permutations, words, matrices

Cited in **2** Documents

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