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Numerical analysis and implementational aspects of a new multilevel grid deformation method. (English) Zbl 1191.65160

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Summary: Recently, we introduced and mathematically analysed a new method for grid deformation [SIAM J. Sci. Comput. 31, No. 2, 1539–1557 (2009; Zbl 1211.65160)] we call basic deformation method (BDM) here. It generalises the method proposed by *G. Liao* et al. [Numer. Methods Partial Differ. Equations 12, No. 4, 489–506 (1996; Zbl 0856.65109); Comput. Math. Appl. 48, No. 7–8, 1077–1085 (2004; Zbl 1066.65097); SIAM J. Sci. Comput. 20, No. 3, 811–825 (1999; Zbl 0929.76091)].

In this article, we employ the BDM as core of a new multilevel deformation method (MDM) which leads to vast improvements regarding robustness, accuracy and speed. We achieve this by splitting up the deformation process in a sequence of easier subproblems and by exploiting grid hierarchy. Being of optimal asymptotic complexity, we experience speed-ups up to a factor of 15 in our test cases compared to the BDM. This gives our MDM the potential for tackling large grids and time-dependent problems, where possibly the grid must be dynamically deformed once per time step according to the user's needs. Moreover, we elaborate on implementational aspects, in particular efficient grid searching, which is a key ingredient of the BDM.

MSC:

65N50 Mesh generation, refinement, and adaptive methods for boundary value problems involving PDEs

Keywords:

grid generation; deformation method; grid adaptation

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