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Classical and quantum dynamics of constrained Hamiltonian systems. (English)

Zbl 1259.70001

Hackensack, NJ: World Scientific (ISBN 978-981-4299-64-0/hbk; 978-981-4299-65-7/ebook). xiv, 302 p. (2010).

This book is an introduction to one of the most interesting topics in theoretical phyics, the study of constrained Hamiltonian systems and their quantization. The topic was introduced by P. G. Bergmann in 1949, and later developed by P.A.M. Dirac, so that it is sometimes known as Dirac-Bergmann theory of constraints. The starting point is that given a Lagrangian L depending on positions and velocities, we cannot develop the Hamiltonian counterpart when L is singular; this means that the Hessian matrix of L with respect to the velocities is not regular, and therefore the accelerations cannot be determined using the well-known Newton equation. In other words, the momenta are not explicitly defined, but implicitly, so that we have a set of primary constraints involving positions, velocities and momenta; these are called primary constraints. In addition, the evolution of primary constraints should preserve them, and introduce new constraints (the so-called secondary constraints). Dirac developed an algorithm precisely just to treat this problem, and at the end he constructed the so-called Dirac bracket using the classification of constraints in first and second class (let us recall that a constraint is named first class if the Poisson bracket with any other constraint vanishes; otherwise is second class). The final purpose is to get a new Poisson bracket (Dirac bracket) in order to quantize singular systems (for instance, electromagnetism); the Dirac bracket is just a modification of the canonical Poisson bracket on the cotangent manifold.

The first seven chapters are devoted to the theory of constraints, including the study of local symmetries, the Dirac conjecture and the Batalin, Fradkin and Tyutin (BFT) formalism, the latter giving a procedure to avoid the problems caused by the existence of second class constraints.

Chapter 8 deals with an important topic, the possibility to develop a Hamilton-Jacobi theory for constrained systems; the authors consider separately the cases of first class systems and second class systems, since the existence of second class constraints presents fundamental problems. We refer here to a recent paper [*M. de León* et al., "On the Hamilton-Jacobi theory for singular Lagrangian systems", Preprint, arXiv:1204.6217] that considers a geometric Hamilton-Jacobi theory for singular Lagrangian systems independently of the class of constraints.

The last four chapters are devoted to the quantization problem, including recent topics, such as the field-antifield formalism of Batalin and Vilkovisky, and a brief discussion of gauge anomalies. The book also contains five appendices that complement the content of the twelve chapters.

The book is well-written and is relevant for theoretical physicists and also for geometers working in mechanics. I would like to call the authors to pay their attention to a remarkable piece of work on the Dirac theory of constraints, that is, its geometrization due to Mark J. Gotay in his PhD thesis, as well as in several papers in collaboration with Hinds and Nester, see [*M. J. Gotay, J. M. Nester* and *G. Hinds*, J. Math. Phys. 19, 2388–2399 (1978; Zbl 0418.58010); *M. J. Gotay* and *J. M. Nester*, Ann. Inst. Henri Poincaré, Nouv. Sér., Sect. A 30, 129-142 (1979; Zbl 0414.58015); ibid. 32, 1–13 (1980; Zbl 0453.58016); "Apartheid in the Dirac theory of constraints", J. Phys. A 17, No. 15, 3063–3066 (1984)].

Gotay's work has influenced several geometers to continue his work; see for instance [*D. Chinea* et al., J. Math. Phys. 35, No. 7, 3410–3447 (1994; Zbl 0810.70014); *A. Ibort* et al., Fortschr. Phys. 47, No. 5, 459–492 (1999; Zbl 0956.37049); *J. F. Cariñena* and *M. F. Rañada*, Lett. Math. Phys. 15, No. 4, 305–311 (1988; Zbl 0655.58028); *M.C. Muñoz Lecanda*, Int. J. Theor. Phys. 28, No. 11, 1405–1417 (1989; Zbl 0697.58019)], among others.

Reviewer: Manuel de León (Madrid)

MSC:

70-02	Research exposition (monographs, survey articles) pertaining to mechanics of particles and systems	Cited in **17** Documents
70H45	Constrained dynamics, Dirac's theory of constraints	
70H20	Hamilton-Jacobi equations in mechanics	

Keywords:

Hamilton-Jacobi equation; quantization; Poisson bracket

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