Caputo, Pietro; **Liggett, Thomas M.**; **Richthammer, Thomas**

Proof of Aldous' spectral gap conjecture. (English) Zbl 1203.60145

J. Am. Math. Soc. 23, No. 3, 831-851 (2010).

Two random processes are considered on an undirected graph with edge weights: the random walk process with the weights as jump rates and the random transposition (or interchange) process with the weights as transition rates. Aldous' spectral gap conjecture asserts that both processes have the same spectral gap. The recursive strategy used to prove the conjecture is a natural extension of the method already used for trees. The novelty is an idea based on electric network reduction, which reduces the problem to the proof of an explicit inequality for a random transposition operator involving both positive and negative rates. The proof of the latter inequality uses coset decompositions of the associated matrices with rows and columns indexed by permutations. In the last section the authors study the spectral gap of other stochastic processes associated to weighted graphs. Symmetric exclusion process, cycle process and matching process are successively considered.

Reviewer: Dominique Lepingle (Orléans)

MSC:

60K35 Interacting random processes; statistical mechanics type models; percolation theory
60J27 Continuous-time Markov processes on discrete state spaces
05C50 Graphs and linear algebra (matrices, eigenvalues, etc.)

Keywords:

random walk; weighted graph; spectral gap; interchange process; symmetric exclusion process

Full Text: DOI arXiv

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