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Invariant Banach limits and applications. (English) Zbl 1205.46012

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Let ℓ_∞ be the space of all bounded sequences $x = (x_1, x_2, \dots)$ with the norm $\|x\|_{\ell_\infty} = \sup_n |x_n|$ and let $L(\ell_\infty)$ be the set of all bounded linear operators on ℓ_∞ . A linear functional B in the dual space ℓ_∞^* is said to be a Banach limit if $B(1, 1, \dots) = 1$, $B \geq 0$ and $B(Tx) = B(x)$ for every $x \in \ell_\infty$, where T , is the translation operator, that is, $T(x_1, x_2, \dots) = (x_2, x_3, \dots)$.

Following an approach similar to that of *W.F. Eberlein* for regular Hausdorff transformations in the classical paper [Proc. Am. Math. Soc. 1, 662–665 (1950; [Zbl 0039.12102](#))] and motivated by their own recent contributions for the dilation operator σ_n , $n \in \mathbb{N}$, and the classical Cesàro operator C , the authors present a set of easily verifiable sufficient conditions on an operator $H \in L(\ell_\infty)$, guaranteeing the existence of a Banach limit B on ℓ_∞ such that $B = BH$. They apply their results to the above mentioned Cesàro operator C on ℓ_∞ and give a necessary and sufficient condition for an element $x \in \ell_\infty$ to have fixed value Bx for all Cesàro invariant Banach limits B . Finally, they apply the preceding description to obtain a characterization of “measurable elements” from the (Dixmier-)Macaev-Sargent ideal of compact operators with respect to an important subclass of Dixmier traces generated by all Cesàro-invariant Banach limits. It is shown that this class is strictly larger than the class of all “measurable elements” with respect to the class of all Dixmier traces.

Reviewer: [Carsten Michels \(Oldenburg\)](#)

MSC:

- [46B99](#) Normed linear spaces and Banach spaces; Banach lattices
- [46L51](#) Noncommutative measure and integration
- [40J05](#) Summability in abstract structures (should also be assigned at least one other classification number from Section 40-XX)

Cited in **19** Documents

Keywords:

[Cesàro-invariant Banach limits](#); [singular traces](#)

Full Text: [DOI](#)

References:

- [1] Banach, S., Théorie des opérations linéaires, (1995), Chelsea Publishing Co. New York, (in French) · [Zbl 0067.08902](#)
- [2] Carey, A.; Phillips, J.; Sukochev, F.A., Spectral flow and Dixmier traces, Adv. math., 173, 68-113, (2003) · [Zbl 1015.19003](#)
- [3] Carey, A.L.; Sukochev, F.A., Dixmier traces and some applications in non-commutative geometry, Russian math. surveys, 61, 1039-1099, (2006) · [Zbl 1151.46053](#)
- [4] Connes, A., Noncommutative geometry, (1994), Academic Press San Diego · [Zbl 1106.58004](#)
- [5] Dodds, P.G.; de Pagter, B.; Sedaev, A.A.; Semenov, E.M.; Sukochev, F.A., Singular symmetric functionals and Banach limits with additional invariance properties, Izv. math., 67, 6, 1187-1212, (2003) · [Zbl 1075.46028](#)
- [6] Dodds, P.G.; de Pagter, B.; Sedaev, A.A.; Semenov, E.M.; Sukochev, F.A., Singular symmetric functionals, J. math. sci., 2, 4867-4885, (2004) · [Zbl 1090.46020](#)
- [7] Dodds, P.G.; de Pagter, B.; Semenov, E.M.; Sukochev, F.A., Symmetric functional and singular traces, Positivity, 2, 1, 47-75, (1998) · [Zbl 0915.46021](#)
- [8] Eberlein, W.F., Banach-Hausdorff limits, Proc. amer. math. soc., 1, 662-665, (1950) · [Zbl 0039.12102](#)
- [9] Lord, S.; Sedaev, A.; Sukochev, F.A., Dixmier traces as singular symmetric functionals and applications to measurable operators, J. funct. anal., 244, 1, 72-106, (2005) · [Zbl 1081.46042](#)
- [10] Lorentz, G.G., A contribution to the theory of divergent sequences, Acta. math., 80, 167-190, (1948) · [Zbl 0031.29501](#)
- [11] Macaev, V.I., A class of completely continuous operators, Dokl. akad. nauk SSSR, 139, 548-551, (1961), (in Russian)
- [12] Pietsch, A., About the Banach envelope of \mathfrak{S}_1 , Rev. mat. complut., 22, 1, 209-226, (2009) · [Zbl 1175.46004](#)

- [13] Raimi, R.A., Factorization of summability-preserving generalized limits, J. lond. math. soc., 22, 398-402, (1980) · [Zbl 0419.40007](#)
- [14] Sargent, W.L.C., Some sequence spaces related to the l_p spaces, J. lond. math. soc., 35, 161-171, (1960) · [Zbl 0090.03703](#)
- [15] Sucheston, L., Banach limits, Amer. math. monthly, 74, 308-311, (1967) · [Zbl 0148.12202](#)

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