W. Ebeling and R. Feistel [VII. Internationale Konferenz über Nichtlineare Schwingungen, Band II, 269-276 (1975)] posed the following mathematical model

\[
\begin{align*}
\dot{x} &= -a_1 x - a_2 x^2 + a_3 x y + a_4 x^2 y, \\
\dot{y} &= a_0 + a_2 x^2 - a_3 x y - a_4 x^2 y
\end{align*}
\]

for a biochemical reaction with second order autocatalysis, where \(a_0 > 0, a_1 > 0\) and \(a_i \geq 0 (i = 2, 3, 4)\) are constants. Using a numerical method, they obtained an example saying that (1) has a limit cycle. Z. J. Jing and L. S. Chen [Acta Math. Appl. Sin. 6, 183-190 (1983; Zbl 0513.34036)] theoretically studied (1) by using qualitative analysis when \(a_3 = 0\). For \(a_3 \neq 0\) in this note the authors obtain the unique existence of limit cycle of (1) when \(B - A^2 > 0\), where \(B = (a_1 a_4 - a_2 a_3)/a_1^2\), \(A = (a_1 a_3 + a_0 a_4)/a_1^2\), and the absence of limit cycles when \(B \leq 0\). The case \(0 < B \leq A^2\) is not discussed.

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MSC:
- 34C05 Topological structure of integral curves, singular points, limit cycles of ordinary differential equations
- 92Cxx Physiological, cellular and medical topics
- 92Exx Chemistry

Keywords:
- biochemical reaction; second order autocatalysis; numerical method; limit cycle