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Symplectic structures, their Bäcklund transformations and hereditary symmetries. (English)

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Summary: It is shown that compatible symplectic structures lead in a natural way to hereditary symmetries. (We recall that a hereditary symmetry is an operator-valued function which immediately yields a hierarchy of evolution equations, each having infinitely many commuting symmetries all generated by this hereditary symmetry. Furthermore this hereditary symmetry usually describes completely the soliton structure and the conservation laws of these equations). This result then provides us with a method for constructing hereditary symmetries and hence exactly solvable evolution equations. In addition, we show how symplectic structures transform under Bäcklund transformations. This leads to a method for generating a whole class of symplectic structures from a given one. Several examples and applications are given illustrating the above results. Also the connection of our results with those of Gel'fand and Dikii, and of Magri is briefly pointed out.

MSC:

37K05 Hamiltonian structures, symmetries, variational principles, conservation laws (MSC2010)

37K35 Lie-Bäcklund and other transformations for infinite-dimensional Hamiltonian and Lagrangian systems

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