Berget, Andrew

Products of linear forms and Tutte polynomials. (English) Zbl 1219.05032

Eur. J. Comb. 31, No. 7, 1924-1935 (2010).

Summary: Let Δ be a finite sequence of n vectors from a vector space over any field. We consider the subspace of $\mathrm{Sym}(V)$ spanned by $\prod_{v\in S} v$, where S is a subsequence of Δ. A result of Orlik and Terao [*P. Orlik* and |it H. Terao, "Commutative algebras for arrangements," Nagoya Math. J. 134, 65–73 (1994; Zbl 0801.05019)] provides a doubly indexed direct sum decomposition of this space. The main theorem is that the resulting Hilbert series is the Tutte polynomial evaluation $T(\Delta; 1+x, y)$. Results of Ardila and Postnikov [*F. Ardila* and |it A. Postnikov, "Combinatorics and geometry of power ideals," Trans. Am. Math. Soc. 362, No. 8, 4357–4384 (2010; Zbl 1226.05019)], Orlik and Terao [loc. cit.], Terao [*H. Terao*, "Algebras generated by reciprocals of linear forms," J. Algebra 250, No. 2, 549–558 (2002; Zbl 1049.13011)] , and Wagner [*D.G. Wagner*, "Algebras related to matroids represented in characteristic zero," Eur. J. Comb. 20, No. 7, 701–711 (1999; Zbl 0996.16027)] are obtained as corollaries.

MSC:

05B35 Combinatorial aspects of matroids and geometric lattices
52B05 Combinatorial properties of polytopes and polyhedra (number of faces, shortest paths, etc.)

Keywords:

direct sum decomposition; Hilbert series; Tutte polynomial evaluation; square free monomials

Full Text: DOI arXiv

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